1770 Card Game Problem

11 October 2019

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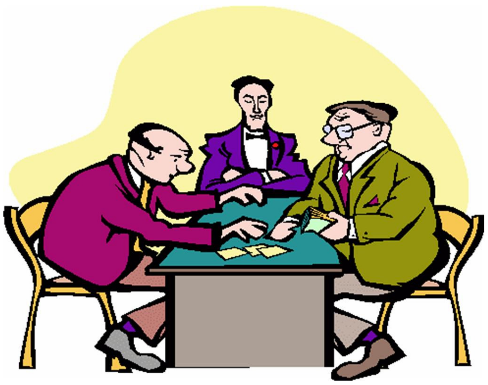

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This problem from the 1987 *Discover* magazine's Brain Bogglers by Michael Stueben ([1]) apparently traces back to 1770, though the exact reference is not given.

Here's an arithmetic problem taken from a textbook published in Germany in 1770. Three people are gambling. In the first game, Player A loses to each of the others as much money as each of them had when the game started. In the next game, B loses to each of the others as much money as each had when that game began. In the third game, A and B each win from C as much money as each had at the start of that game. The players now find that each has the same sum, 24 guineas. How much money did each have when play began?

My Solution

My approach was the straight-forward, pedestrian one. Namely, I wrote down the results of each game and came up with equations to solve for the starting amounts of money each player had. The following table shows the steps.

Players	**A**	**B**	**C**
Starting Amount	x	y	z
End 1st Game	$x-(y+z)=$ $\mathbf{x-y-z}$	$y+y=$ $\mathbf{2y}$	$z+z=$ $\mathbf{2z}$
End 2nd Game	$(x-y-z)+$ $(x-y-z)=$ $\mathbf{2(x-y-z)}$	$2y-(x-y-z)-2z=$ $\mathbf{3y-x-z}$	$2z+2z=$ $\mathbf{4z}$
End 3rd Game	$2(x-y-z)+$ $2(x-y-z)=$ $\mathbf{4x-4y-4z=24}$	$(3y-x-z)+$ $(3y-x-z)=$ $\mathbf{-2x+6y-2z=24}$	$4z-2(x-y-z)-(3y-x-z)=$ $\mathbf{-x-y+7z=24}$

Now we have three linear equations in three unknowns, which can easily be solved:

$$\begin{array}{r} 4x-4y-4z=24 \\ -2x+6y-2z=24 \\ -x-y+7z=24 \end{array} \Rightarrow \begin{array}{r} x-y-z=6 \\ -x+3y-z=12 \\ -x-y+7z=24 \end{array} \Rightarrow \begin{array}{r} 2y-2z=18 \\ -2y+6z=30 \end{array} \Rightarrow 4z=48$$

Therefore, $z = 12$, $y = 21$, and $x = 39$. So A started with 39 guineas, B with 21 guineas, and C with 12 guineas.

Brain Bogglers Solution

A had 39 guineas; B, 21; and C, 12. The trick is to work backwards. Since A and B doubled their money to 24 guineas in the last round, the three players must have entered the round with the sums: A, 10; B, 12; C, 48. In the second game, A and C doubled their money, so before the second game the tally stood at A, 6; B, 42; C, 24. In the first game, B and C doubled their money, so at the start the players must have had the amounts listed above.

The Brain Bogglers solution is clearly simpler and more elegant (sigh).

References

[1] Stueben, Michael, "Brain Bogglers," *Discover*, March 1987