

Kissing Angles

6 May 2019

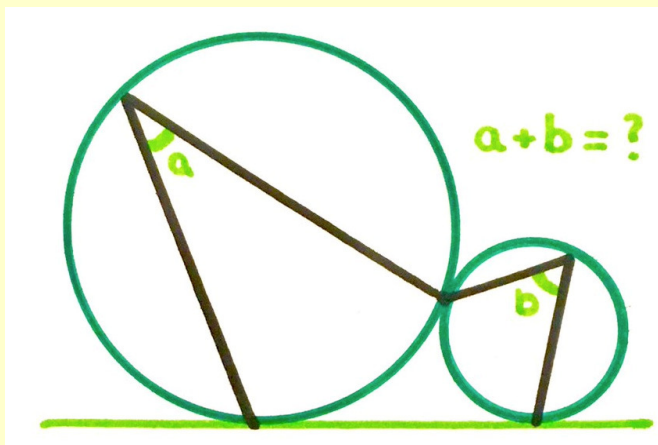
Jim Stevenson

I really was trying to stop including Catriona Shearer's problems, since they are probably all well-known and popular by now. But this is another virtually one-step-solution problem that again seems impossible at first. Many of her problems entail more steps, but I am especially intrigued by the one-step problems.

Catriona Shearer @Cshearer41 3 May 2019

(<https://twitter.com/Cshearer41/status/1124319237160472579>)

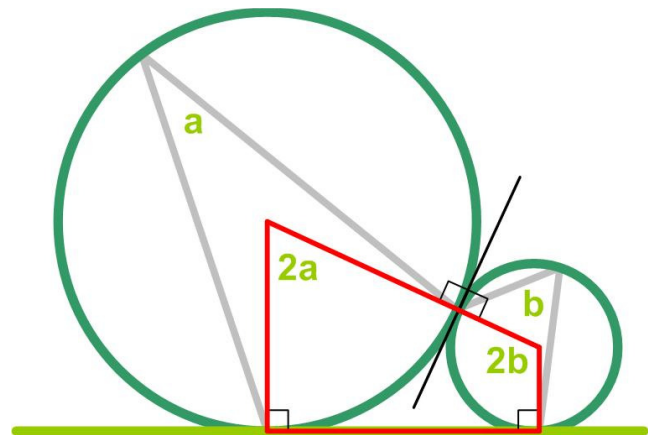
What's the sum of the two marked angles?



Solution

Once you add the central angles corresponding to the angles inscribed in the circles, the solution becomes evident.

A little bit of argument is required to verify that the line joining the centers of the two circles passes through the tangent point. But since the two circles are tangent to each other, they share a common tangent line, as shown. But this line is also perpendicular to each of the radii from the centers of the circles, and so the radii make an angle of $90^\circ + 90^\circ = 180^\circ$ with each other, namely a straight line.



Therefore, $2a + 2b = 180^\circ$, since the other angles in the quadrilateral add up to 180° (or the sum of the angles indicated when a transverse line crosses two parallel lines is 180°). And so $a + b = 90^\circ$.

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