Two Trains – Passing in the Night

27 January 2019

Jim Stevenson

This is one of H. E. Dudeney's train puzzles ([1] p.19).

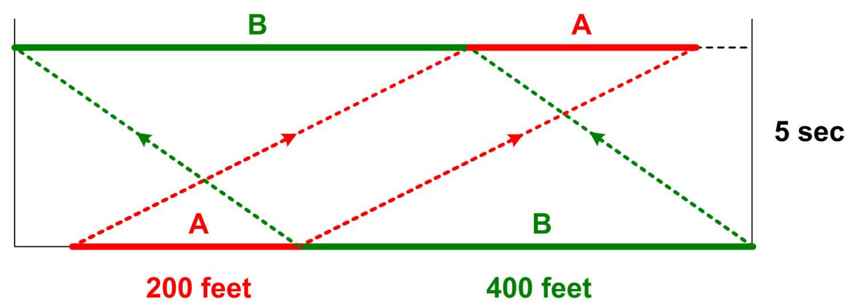
67. TWO TRAINS – PASSING IN THE NIGHT

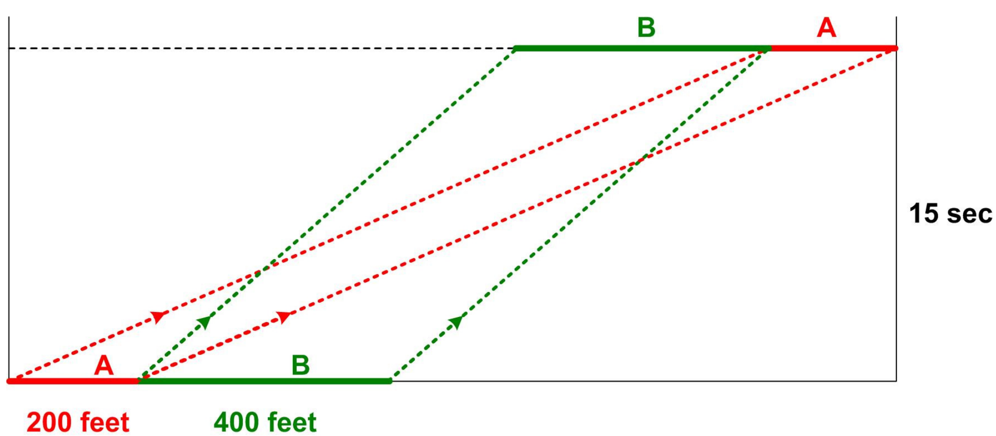
Two railway trains, one four hundred feet long and the other two hundred feet long, ran on parallel rails. It was found that when they went in opposite directions they passed each other in five seconds, but when they ran in the same direction the faster train would pass the other in fifteen seconds. A curious passenger worked out from these facts the rate per hour at which each train ran.

Can the reader discover the correct answer? Of course, each train ran with a uniform velocity.

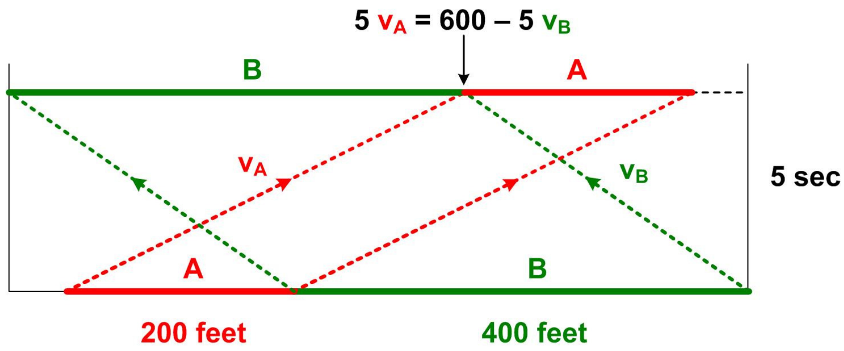
Solution

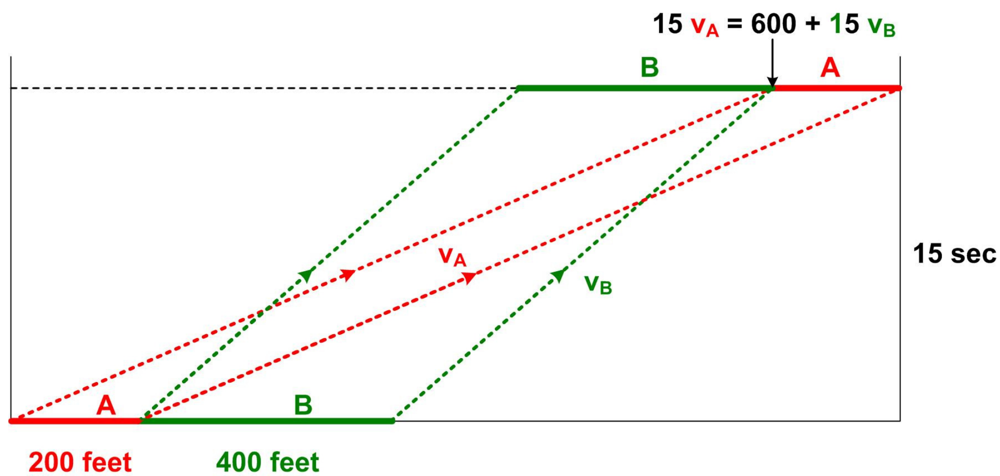
The biggest challenge for this problem is setting up the models:





Once this is done, the solution is fairly straight-forward. Let v_A be the speed of the 200 ft train and v_B the speed of the 400 ft train. Then as we see from the diagrams, $5\ v_A = 600 - 5\ v_B$ and $15\ v_A = 600 + 15\ v_B$, which implies that $v_A = 120 - v_B$ and $v_A = 40 + v_B$, so that $v_A = 80$ft/sec, $v_A = 40$ft/sec.





Now all we need to do is convert from ft/sec to mph.

$$v_A = 80\ \frac{\text{feet}}{\text{sec}}\ \frac{60\text{ sec}}{1\text{ min}}\ \frac{60\text{ min}}{1\text{hr}}\ \frac{1\text{ mi}}{5280\text{ ft}} = 80\ \frac{3600\text{ mi}}{5280\text{ hr}} = 53\ {}^{6}/_{11}\text{ mph}$$

$v_B = 27\ {}^{3}/_{11}$ ft/sec

References

[1] Dudeney, Henry Ernest, *536 Puzzles & Curious Problems*, Edited By Martin Gardner, Charles Scribner's Sons, New York, 1967.