

Parallelogram Problem

(23 October 2018)

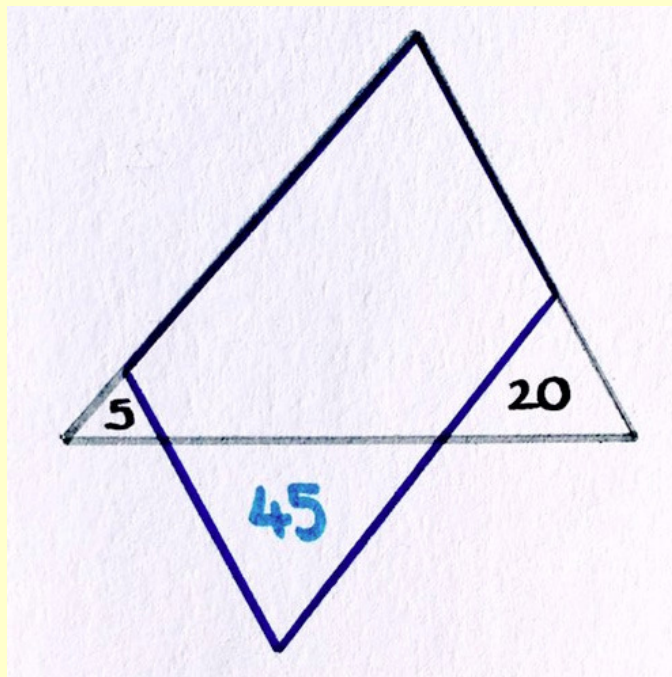
Jim Stevenson

Catriona Shearer has come up with another challenging but elegant geometric problem. In some ways, it is similar to the famous Russian Coffin Problems that have an obvious solution—once you see it—but initially seem impenetrable. I really marvel at Catriona Shearer's ability to come up with these problems. They seem to be garnering a great deal of attention on the internet, as well they should. They show a remarkable imagination with their clean, economical presentation.

Catriona Shearer (<https://twitter.com/Cshearer41>) October 21, 2018

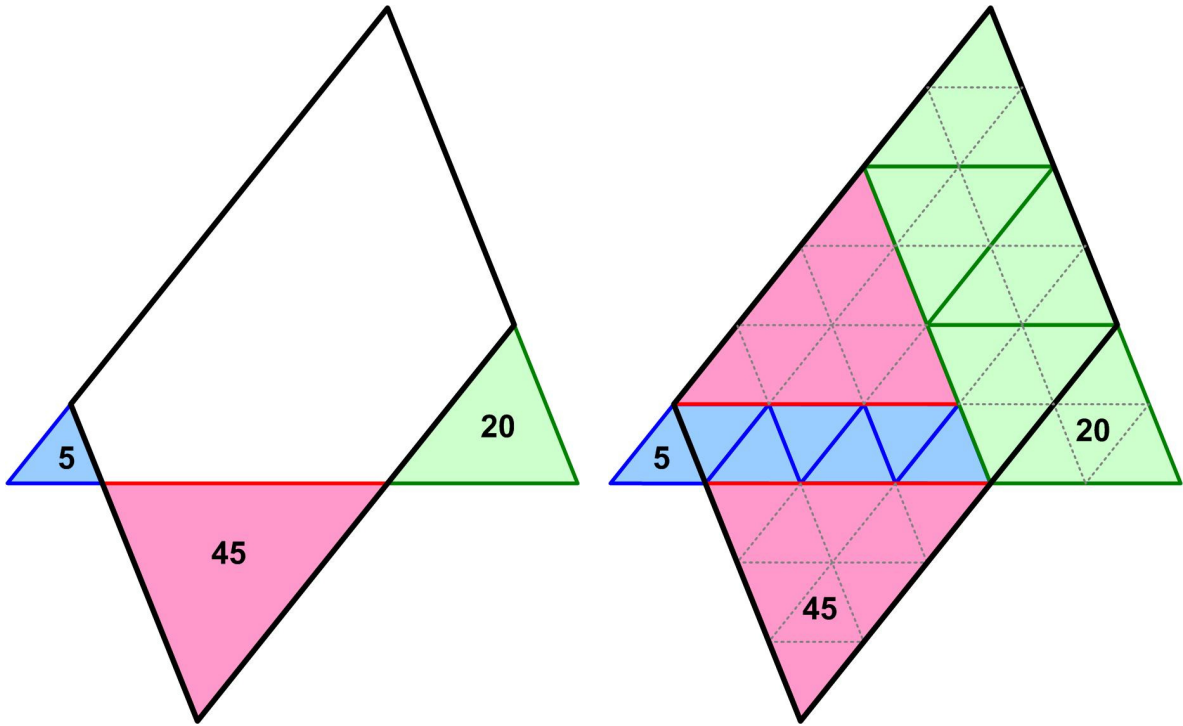
(<https://twitter.com/Cshearer41/status/1054024681622327296>, retrieved 10/22/2018)

What's the area of the parallelogram?



Solution

The answer is 200. I will save the revealing picture for the next page, since once you see it, the result is obvious.



There are a couple of keys to the problem. First is the obvious fact (that took me a while to realize) that the 45 and 20 triangles are made up of integral multiples of the 5 triangle, which is really the heart of the solution. The other relevant fact is that the long side of the parallelogram is made up of the edges of the 45 and 20 triangles and the short side is made up of the edges of the 5 and 45 triangles. These two facts guarantee an integral number of 5 triangles will lie along each side of the parallelogram.

So one approach is to see that the parallelogram is made up of 4 of the 20 triangles, 2 of the 45 triangle and 6 of the 5 triangles. This gives the area of 200. Alternatively, we can see there are 4 rows of the 5 triangles in the parallelogram, each consisting of 10 triangles, or $40 \times 5 = 200$.

