More Sum of Angles

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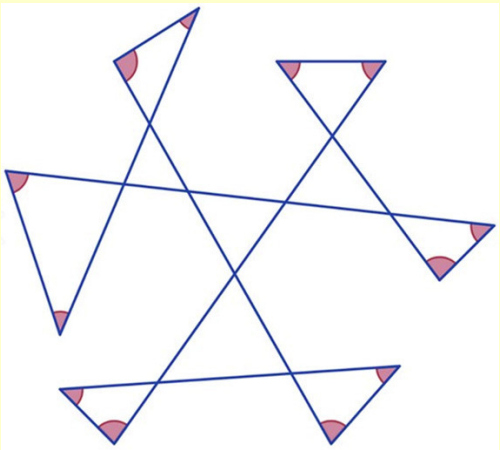
Mathnasium of Amarillo (https://twitter.com/mathnasiumAMA) had a nice follow up on the sum of angles type of puzzle.

(https://twitter.com/mathnasiumAMA/status/851568014671699968)

10 Apr 2017

Find the total value of pink colored angles.



Solution

We proceed as in the *Star Sum of Angles* problem ([1]). Label the relevant angles a, b, …, k, l and the sum of these angles S = a + b + … + k + l. As shown in Figure 1 start with a (red) arrow at

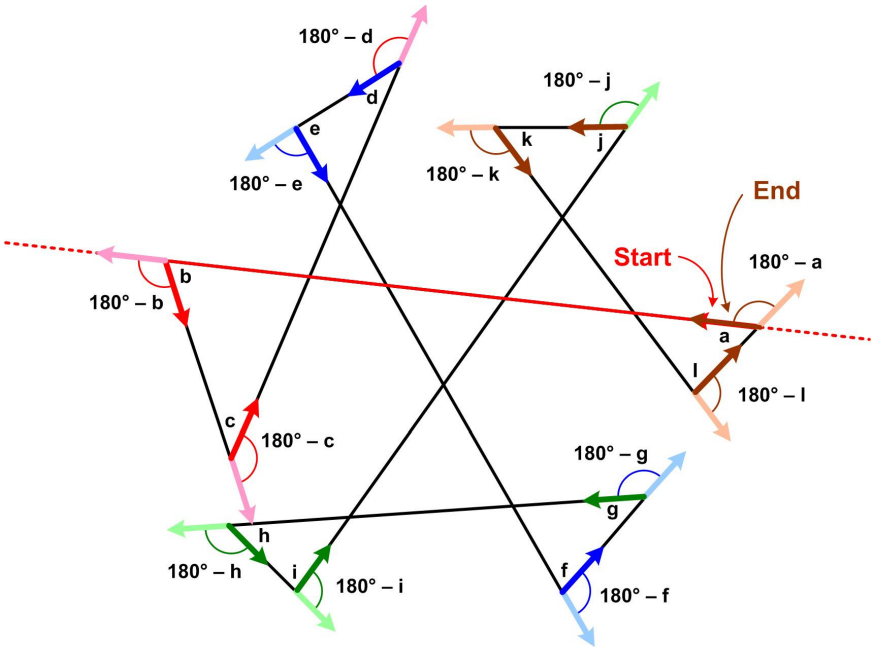


Figure 1

the vertex of angle a. Slide the arrow along the initial (red) edge of the figure to the vertex at angle b and rotate it until it lies along the adjacent edge to angle b. Then slide the arrow to the vertex at angle c, and so forth, until the arrow arrives back at the starting position at the vertex of angle a. Each time the arrow rotates past the angle of the initial red line, that constitutes one revolution of 360°. I have changed the color of the arrow in the figure to indicate when a full revolution has occurred.

That means the sum of integral revolutions of 360° (since the arrow returns to its original position) equals the sum of the angles swept out by the rotating arrow, or

$$4 \cdot 360° = (180° - a) + (180° - b) + \ldots + (180° - k) + (180° - l) = 12 \cdot 180° - S$$

or

$$S = (12 - 8) \cdot 180° = 720°$$

Alternative Solution

A contributor to Mathnasium of Amarillo's twitter post on 23 November 2018, Snail Erato (https://twitter.com/EratoSnail), besides giving my solution above, also provided a second solution that worked in this special case where the enclosed figures are all triangles (https://twitter.com/EratoSnail/status/1065989272551993344).

Color the triangles as shown in Figure 2. Then S = (Sum of angles in blue triangles) – (Sum of angles in red triangles) + (Sum of angles in the green triangle), or

$$S = 6 \cdot 180° - 3 \cdot 180° + 1 \cdot 180°$$

or

$$S = 4 \cdot 180° = 720°$$

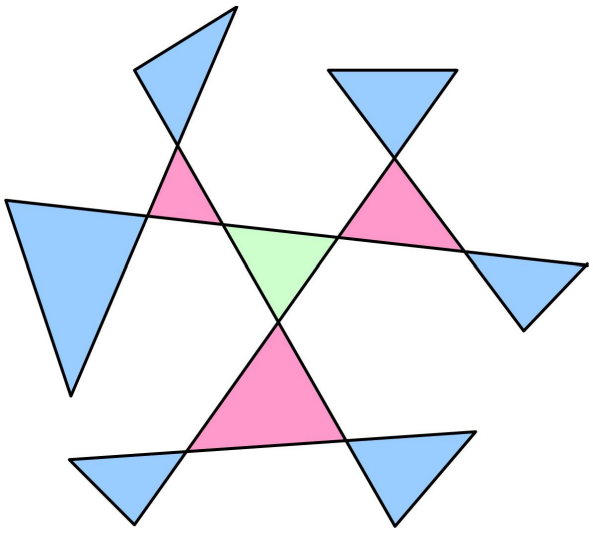


Figure 2 Alternative Solution

References

[1] Stevenson, James, "Star Sum of Angles," in *Meditations on Mathematics*, 7 December 2018 (http://josmfs.net/2019/02/12/star-sum-of-angles/)