Chalkdust Grid Problem

(16 October 2018)

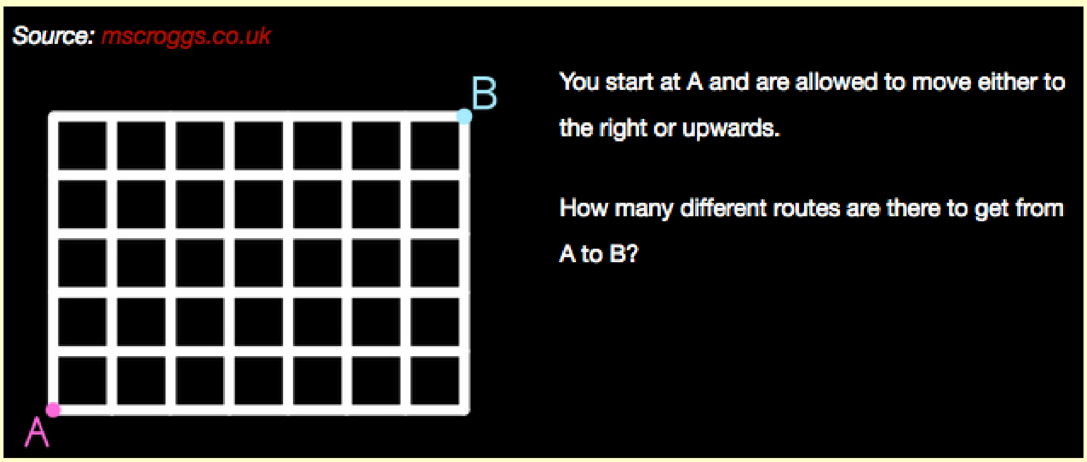
Jim Stevenson

Normally I don't care for combinatorial problems, but this problem seemed to bug me enough to try to solve it. It took me a while to see the proper pattern, and then it was rather satisfying.

chalkdust

A magazine for the mathematically curious

(http://chalkdustmagazine.com/regulars/puzzles/puzzles-issue-03/, retrieved 10/11/2018)



(I will pause here before offering my solution)

Solution

Clearly, some type of recursive approach is needed. So we begin at the endpoint, B, and consider its adjacent nodes (Figure 1). Only one path is possible from each of these nodes to B. (We shall arbitrarily say there is 1 path from node B to B.)

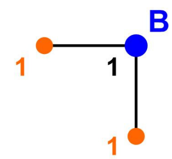


Figure 1
One level back

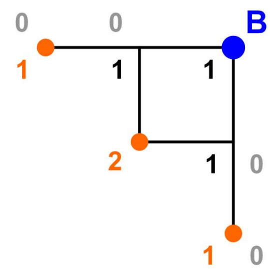


Figure 2
Two levels back

Next consider the three nodes two levels back (Figure 2), adjacent to the one-level back nodes. Only one path exits from each of the outer nodes to B. We see that two paths exit the center node to B. Another way of looking at this is to consider the two adjacent nodes to the center node (via the exiting horizontal and vertical path legs). The number of paths exiting the center node will be the sum of the paths exiting the two adjacent nodes, in this case 1 + 1 = 2. We shall assume 0s are assigned to the nodes exterior to the grid, so that we can extend the recursive sums to all nodes (except B).

Next we consider the number of paths four levels back (Figure 3). Again we see that only one path is possible from each of the outer nodes to B. But 4 paths exit the next node down to B since one path exits the vertically adjacent node and 3 paths exit the horizontally adjacent node, so that 1 + 3 = 4. Similarly the center node has 6 paths exiting it to B, since its two adjacent nodes each have 3 paths to B, so that 3 + 3 = 6.

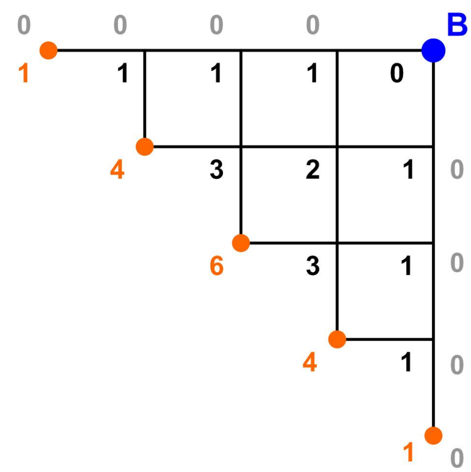


Figure 3 Four levels back

Therefore the number of paths leaving a node to arrive at B equals the sum of the paths leaving the two adjacent (horizontal and vertical) nodes. That is exactly the pattern that defines Pascal's Triangle.

And so we have the final pattern we need (Figure 4). We see that the last node is our node A. Therefore the total number of paths from A to B is 792.

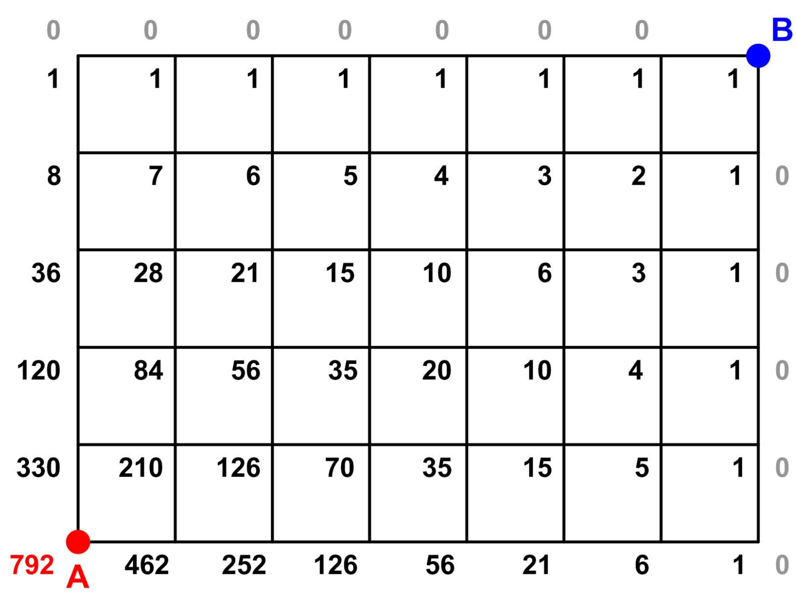


Figure 4 Final solution