Parallelogram Puzzle

(11 August 2016)

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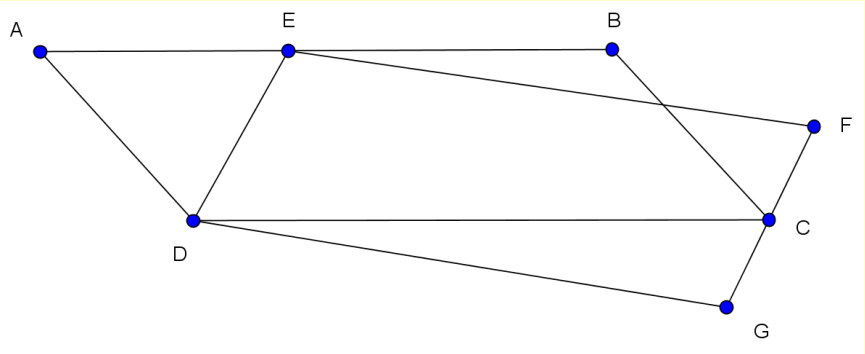
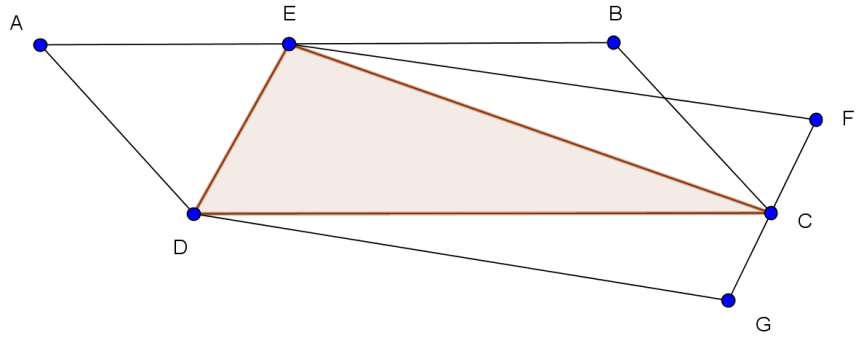
(http://www.futilitycloset.com/2016/08/10/parallelogram-puzzle/, retrieved 8/11/16)



Point E lies on segment AB, and point C lies on segment FG. The area of parallelogram ABCD is 20 square units. What's the area of parallelogram EFGD?

Futility Closet Answer



Draw EC. Now parallelogram ABCD and triangle EDC share a common base (DC), and they have the same altitude (a perpendicular from E to DC). So triangle EDC has half the area of parallelogram ABCD.

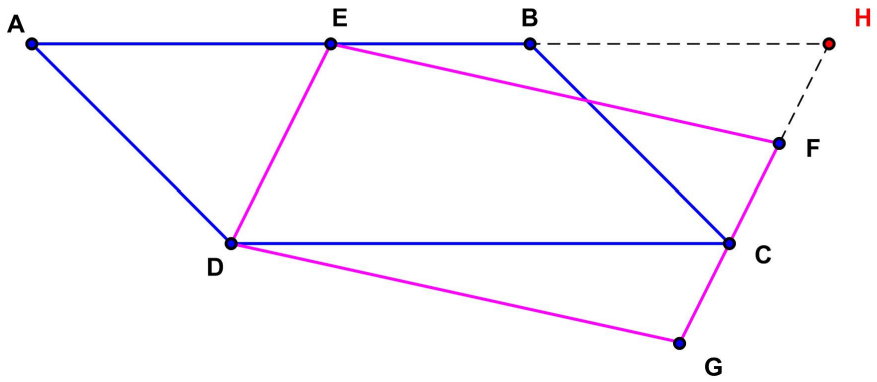
But likewise, parallelogram EFGD and triangle EDC share a common base (ED), and they have the same altitude (a perpendicular from C to ED). So triangle EDC has half the area of parallelogram EFGD.

Since both parallelograms have twice the area of the same triangle, their own areas must be equal. So the area of EFGD is 20 square units.

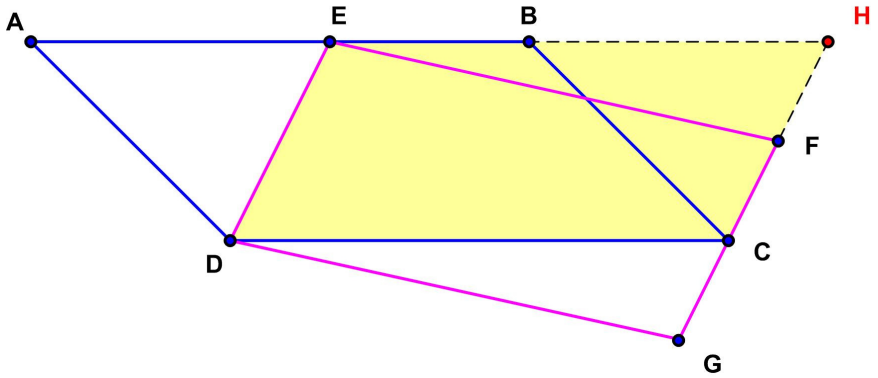
(From Alfred S. Posamentier, *Math Wonders to Inspire Teachers and Students*, 2003.)

My Solution

My approach is similar in that it requires adding some lines to the figure, namely extend the top edge of the blue parallelogram to meet the extension of the right edge of the pink parallelogram at the vertex H.



This defines a new (yellow) parallelogram EHCD (the intersection points of two pairs of intersecting parallel lines define a parallelogram). Since this parallelogram has the same base and altitude as the blue and the pink parallelograms, it has the same area. So the original parallelograms have the same area, which means the area of the pink parallelogram is also 20 square units.



I prefer this solution since I believe it is visually easier to see all the parallelograms have the same area. For example, just sheer the blue parallelogram to the right to coincide with the yellow one. And then sheer the yellow one down to coincide with the pink one.