

# A Multitude of 2s

31 January 2019

Jim Stevenson



This is a fun little problem from the United Kingdom Mathematics Trust (UKMT) Senior Math Challenge of 2008.

“What is the remainder when the 2008-digit number 222 ... 22 is divided by 9?”

(Hint: See The Barrel of Beer)

## Solution

In the solution to the Barrel of Beer problem we introduced the idea of the digital root and modular arithmetic with the mod 9 function  $((x)_9)$ . The essence of the digital root idea is to convert a decimal number into its sum of digits. So

$$\begin{aligned}(222 \dots 22)_9 &= \underbrace{((2)_9 + (2)_9 + (2)_9 + \dots + (2)_9 + (2)_9)}_{2008}_9 \\ &= (2008 \cdot (2)_9)_9 = ((2008)_9 \cdot (2)_9)_9 = ((10)_9 \cdot (2)_9)_9 = 1 \cdot 2 \\ &= 2\end{aligned}$$

(c) 2019 James Stevenson

---