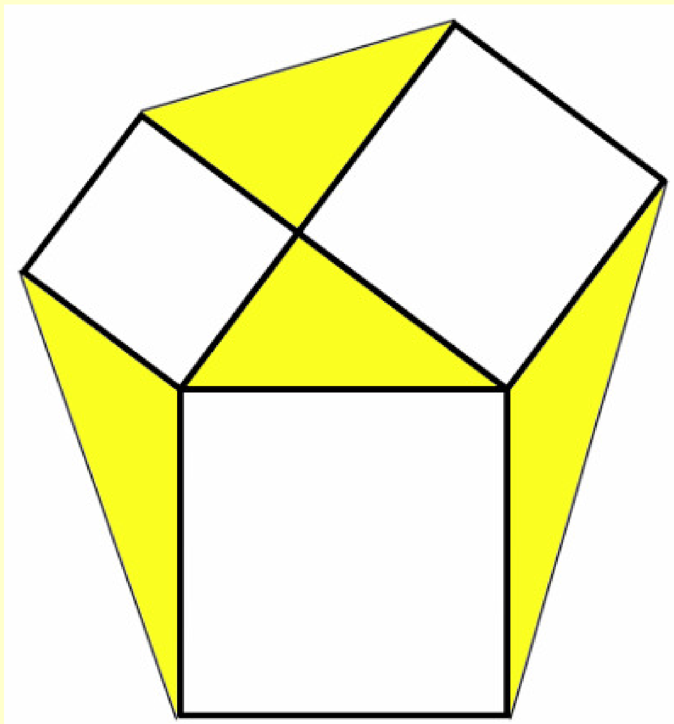
Four of a Kind

(11 February 2018)

Jim Stevenson

From Futility Closet we have another intriguing problem with what turns out to be a simple and elegant solution.

(https://www.futilitycloset.com/2018/02/11/four-of-a-kind/, retrieved 2/12/2018)



If squares are drawn on the sides of a triangle and external to it, then the areas of the triangles formed between the squares all [each] equal the area of the triangle itself.

(Roger Webster, "Bride's Chair Revisited," *Mathematical Gazette* 78:483 [November 1994], 345-346.)

Solution

I originally assumed that the center triangle was a right triangle as suggested by the picture. But then I realized there was a solution that did not depend on that. All that was required was that the three quadrilaterals be squares touching at their vertices. Therefore the center triangle can be any type of triangle. The solution is shown in Figure 1.

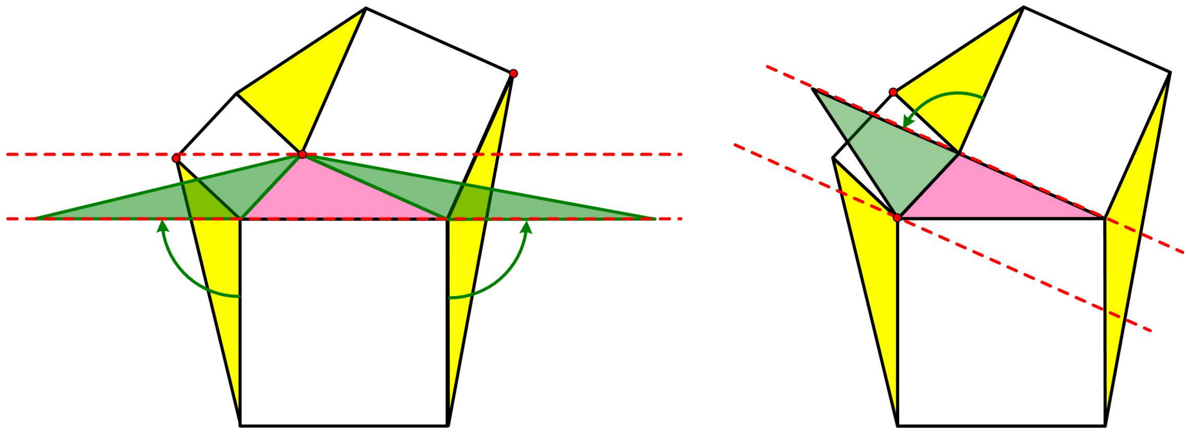


Figure 1 Rotating each yellow triangle 90º yields a (green) triangle with same base and altitude as the center (pink) triangle, and thus the same area

The details of the proof are left to the reader. That is, one needs to verify that the rotated green triangles lie on the same straight line as the pink triangle and that the vertex of each green triangle opposite the base coincides with the similar vertex of the pink triangle (so that it has the same altitude). Notice that the green and red bases are the same length since they were the sides of the same square.