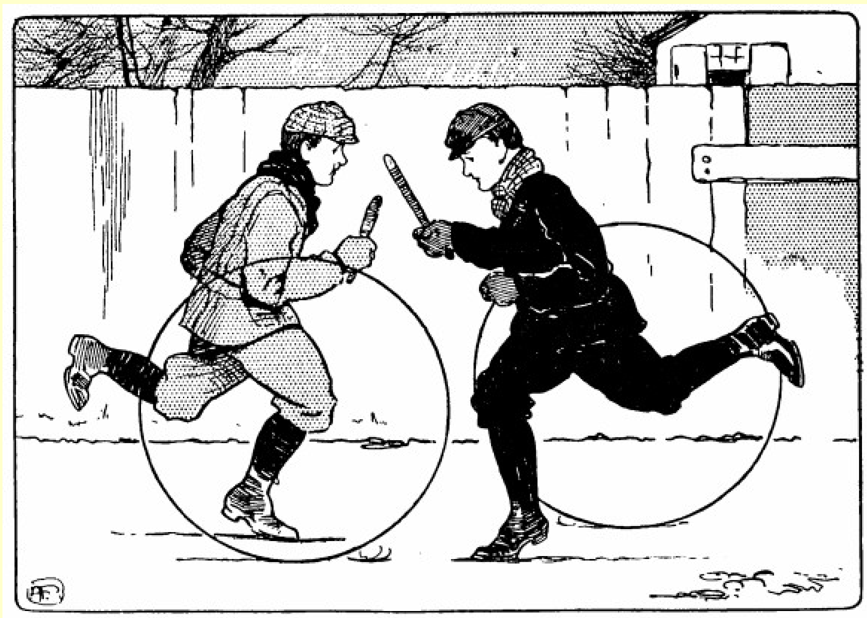
The Two Errand Boys

(21 December 2016)

Jim Stevenson

(https://www.futilitycloset.com/2016/12/19/two-errand-boys/, retrieved 12/21/2016)



Another conundrum from Henry Dudeney's *Canterbury Puzzles*:

A country baker sent off his boy with a message to the butcher in the next village, and at the same time the butcher sent his boy to the baker. One ran faster than the other, and they were seen to pass at a spot 720 yards from the baker's shop. Each stopped ten minutes at his destination and then started on the return journey, when it was found that they passed each other at a spot 400 yards from the butcher's. How far apart are the two tradesmen's shops? Of course each boy went at a uniform pace throughout.

Futility Closet Solution

All that is necessary is to add the two distances at which they meet to twice their difference. Thus 720 + 400 + 640 = 1760 yards, or one mile, which is the distance required.

A more general solution can be found on page 416 of *Pi Mu Epsilon Journal*, Spring 1982 (PDF[1]).

My Solution

I solved the problem before looking at the Futility Closet Solution. Even after looking at their solution, it did not seem clear how they arrived at it. At least my approach is clearly derived.

I took a direct approach where I worked out expressions relating the rates, distances, and times for each boy as given in the problem. This is illustrated in Figure 1. The distance each boy traveled

[1] http://www.pme-math.org/journal/issues/PMEJ.Vol.7.No.6.pdf [See below p.3]

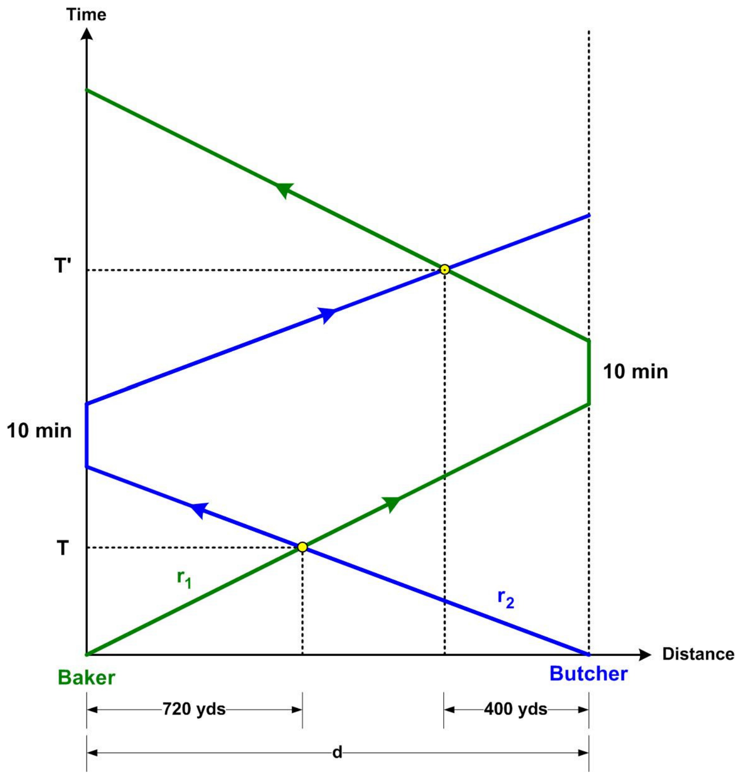


Figure 1 "World lines" for the Two Errand Boys

is plotted horizontally and the time it took to make their errands is plotted vertically. The distance between the Baker and the Butcher that we want to find is given by d.

The points where the boys meet are shown as yellow dots. The speed of the (green) boy starting from the Baker is r_1 and the speed of the (blue) boy starting from the Butcher is r_2, both in yds/min. The boys are assumed to start simultaneously at time = 0 and first meet at time = T minutes. The second time they meet is time = T' minutes. We need only consider the time the boys were actually traveling and ignore the 10 minutes each boy waited at their respective destinations. Visually in Figure 1 we can slide the upper crossing paths down, eliminating the 10 minute gaps, without changing any of the distance relationships.

In any case the equations derived from the problem from the first meeting are

$$720 = r_1\, T \quad \text{and} \quad d - 720 = r_2\, T \tag{1}$$

and from their second meeting are

$$d + 400 = r_1\, (T' - 10) \quad \text{and} \quad d + (d - 400) = r_2\, (T' - 10). \tag{2}$$

From equation (1), we get the two speeds in terms of the time T :

$$r_1 = 720 / T \quad \text{and} \quad r_2 = d - 720 / T$$

Substituting these speeds into equations (2) gives:

$$d + 400 = 720\, (T' - 10) / T \quad \text{and} \quad 2d - 400 = (d - 720)\, (T' - 10) / T.$$

Eliminating (T' – 10) / T from both equations yields the equation

$$2d - 400 = (d - 720)\, (d + 400) / 720.$$

$$2 \cdot 720\, d - 400 \cdot 720 = d^2 + (400 - 720)d - 400 \cdot 720$$

$$d^2 + (400 - 3 \cdot 720)d = 0$$

d = 3·720 – 400 = 2160 – 400 = **1760 yds**.

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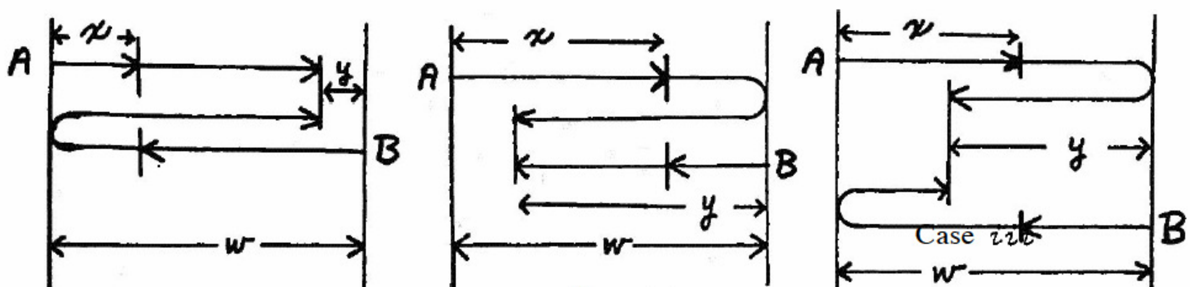
page 416 of *Pi Mu Epsilon Journal*, Spring 1982

486. [Spring 1981] *Proposed by Chuck Allison and Peter Chu, San Pedro, California.*

Swimmers A and B start from opposite sides of a river and swim to their corresponding opposite sides and then back again, each swimming at his own constant rate. If on the first pass they meet each other x feet from A's starting side, and on the second pass they meet at a point y feet from B's starting side, how wide is the river in terms of x and y?

Solution by Kevin Theall, Essex Falls, New Jersey.

Consider the following three cases, shown in the diagrams below.



Case *ii*

Letting v_a and v_b be the respective swimming rates for A and B, we have case *i* when $2v_a \leq v_b$. Then, considering the distances travelled between meetings, we have

$$\frac{v_b}{v_a} = \frac{w - x}{x} = \frac{w - y + x}{w - x - y},$$

whence

$$w = \frac{3x + y + \sqrt{(x-y)^2 + 8x^2}}{2}.$$

If $v_a \geq 2v_b$, then case *ii* occurs and we have

$$\frac{v_b}{v_a} = \frac{w - x}{x} = \frac{y}{w + y}, \text{ so } w = \frac{x - y + \sqrt{(x-y)^2 + 8yx}}{2}$$

Finally, case *iii* occurs when $4v_a > 2v_b > v_a$. Then

$$\frac{v_b}{v_a} = \frac{w - x}{x} = \frac{2w - y}{w + y}, \qquad w = 3x - y.$$

Case *iii* was also solved by LEONOR M. ABRAIDO-FANDIÑO, ARA BASMAKIAN, MIKE BEACH, DAVID DEL SESTO (cases *ii* and *Hi)*, MARK EVANS (who remarked that the quicker swimmer must not be more than twice as fast as the slower swimmer), VICTOR G. FESER (who recognized a second possibility), ROBERT C. GEBHARDT, JOHN M. HOWELL, RALPH KING, HENRY S. LIEBERMAN, BOB PRIELIPP, DOUGLAS RALL (cases A. and *iii*), ANITA REED, KENNETH M. WILKE, BRENT WRASMAN (cases *ii* and *iii*), and the. PROPOSER.