Ancestors Problem

(20 August 2011, rev 30 December 2015)

Jim Stevenson

Going through some old clippings, I came across an article that got excited about how many ancestors we should have at different points in the past. The article is amusing and the possibilities are intriguing. Unfortunately, the methods I considered to solve the questions raised by the article were not that elegant, but they did pose a mental challenge to organize things carefully to see what the answers should be.

The next two pages contain the article and my questions (I substituted a color picture for the original black and white one). The last page has my solution.

The Washington Post

9 September 1978

Michelangelo's 'Creation of Eve'

And 'Roots' Begat ...

The Long, Crowded Road Into the Past

By Henry Allen

The riddle arose when the first showing of Roots-rerun on TV this week topped even Howard Hughes' will for prompting Americans to trace their genealogies.

A writer named Edward Devol, said in March, 1977, that it's fine to go rooting around for ancestors, say 10 generations ago, but keep in mind that back then, you had 1,024 direct ancestors. (You had two parents, four grandparents. Keep multiplying.)

My mistake was to keep multiplying, one rainy Saturday morning. I discovered that; allowing 25 years per generation, I had 4,096 ancestors when the pilgrims landed; a million before Columbus sailed, and a billion not long ago after the Norman Conquest.

The implications were astonishing, especially since four billion people are alive now. The world, it seemed, once had an infinite population, decreasing by half every 25 years. If true, this

meant bad things for the Washington real estate boom.

If false—well, how could it be? Didn't everybody have two parents? And didn't each parent have two parents? The only exception would seem to be virgin births.

I bludgeoned the life out of a couple of 1977 dinner parties with my theory, decided I was ahead of my time and shut up.

Then came the Roots rerun, this week. I decided to get expert advice.

I called Dave Gwatkin, who does international research in population and health, here in Washington.

"Something's got to be wrong," he said, "but I didn't see the program."

So I called Ellen Jamison, an international demographer at the Census Bureau, and explained the problem.

"Somehow that doesn't sound quite right," she said. "Think of it not as going backwards as going forwards. Suppose Adam and Eve have two children, and each of them has two ... "

"Fine," I said. "But just to make the problem simple I assumed I was the only human alive. Imagine how many people there used to be if you start with four billion."

"This really isn't my field," she said. "Maybe you should talk with Eduardo Arriaga."

"You have to be doing something wrong," said Arriaga, who is chief of the evaluation division at the Census, Bureau. "For one thing, you have to remember that several children can have two parents. Say there are four children of two parents. Well, then, each has half a parent. If you have eight, then each has one-quarter of a parent."

"Ahhhh," I said. closing for the pounce. "That would imply, however, that the more children we breed, the smaller the number of people in the past."

"Exactly," Arriaga said, his voice cracking with triumph, like a double-jumping checker. "Compare two populations of 1,000 each. One grows at 3 percent, the other at 1 percent. A year ago, the faster growing one had only 970 people, while the slower one had 990 people."

"Yes," I said. Anything to make him stop it before circuit breakers started slapping shut inside my head.

I called Georgetown University, next, and interrogated Murray Gendell, director of the university's demography program.

"I wish I could give you an answer right off the bat," he said. "But I think the answer lies in mortality rates. Let me check the library."

The next morning his enthusiasm sizzled through the receiver.

"For most of man's history, death rates have nearly balanced birth rates," he said. "The rate of growth in population was exceedingly small, until relatively recently, about 15 per million per year."

Inspiration hit again. "The problem," I said, "is that all of my ancestors had to survive long enough to reproduce, or I wouldn't be here. So death rates don't matter."

At this point, I began to see the possibility of a Nobel in science.

Then I went too far.

I called Herman Chernoff, professor of statistics at MIT. He ended it quickly.

"Incest," Chernoff said.

This terrible thought had flickered at the outer edges of my thinking. In fact, I'd even tried to draw what a family tree would look like if everyone was the product of the union of first cousins. What I found was that these people would have six great-grandparents instead of eight. The progression would be slower—I'd have 20 ancestors rather than 1,024, 10 generations back—but it was still increasing.

I tried this gambit on Chernoff.

"Look at it from the reverse point of view, he said. "We start off with two humans, say Adam and Eve. Assume six children per generation. After 10 generations, you'd have a lot of people. If one of the people in the 10th generation looked back, using your system, he'd think he had 1,024 ancestors 10 generations ago, but he'd be wrong."

"Right. But it's entirely possible that I had two completely unrelated parents, and four unrelated grandparents . . ."

"Sure, Chernoff agreed, with short cheer. "But if you keep multiplying, sooner or later some of those people have to be related. And that means that the increase of ancestors gets smaller, until they begin to decrease all the way back to two."

"Assuming there was not once an infinite number of people in the world?"

"Of course," Chernoff snapped.

My confidence, like that of an overmatched young boxer, had been shattered.

I'd like to think that I've given the world a system for proving you're descended from Aristotle, Attila the Hun, anybody you damn well please. (My personal favorite, among the billions, is Ethelred the Unready, an Anglo-Saxon king.)

But if you use it, you'll have to do the math yourself. And stay away from Chernoff.

1. Do you agree with the author's numbers in this paragraph?
2. What is the minimal set of ancestors a person can have at generation 10 in the past if the person is generation 0 and *first* cousins can marry and there is no incest (parents cannot marry children and siblings cannot marry)?
3. What is the answer to #2 if first cousins cannot marry each other (or their grandparents), but *second* cousins can marry each other?

My solution is on the next page.

Solution:

Figure 1 provides the answers to the questions, and even shows the case where all marriages are between siblings (incest).

Question 1: The author is clearly wrong about marriages between first cousins yielding 6 grandparents as a minimum. The second pattern in Figure 1 shows the minimum to be 4 grandparents. This persists through generation 10, so that the author would still have 4 ancestors then instead of 20.

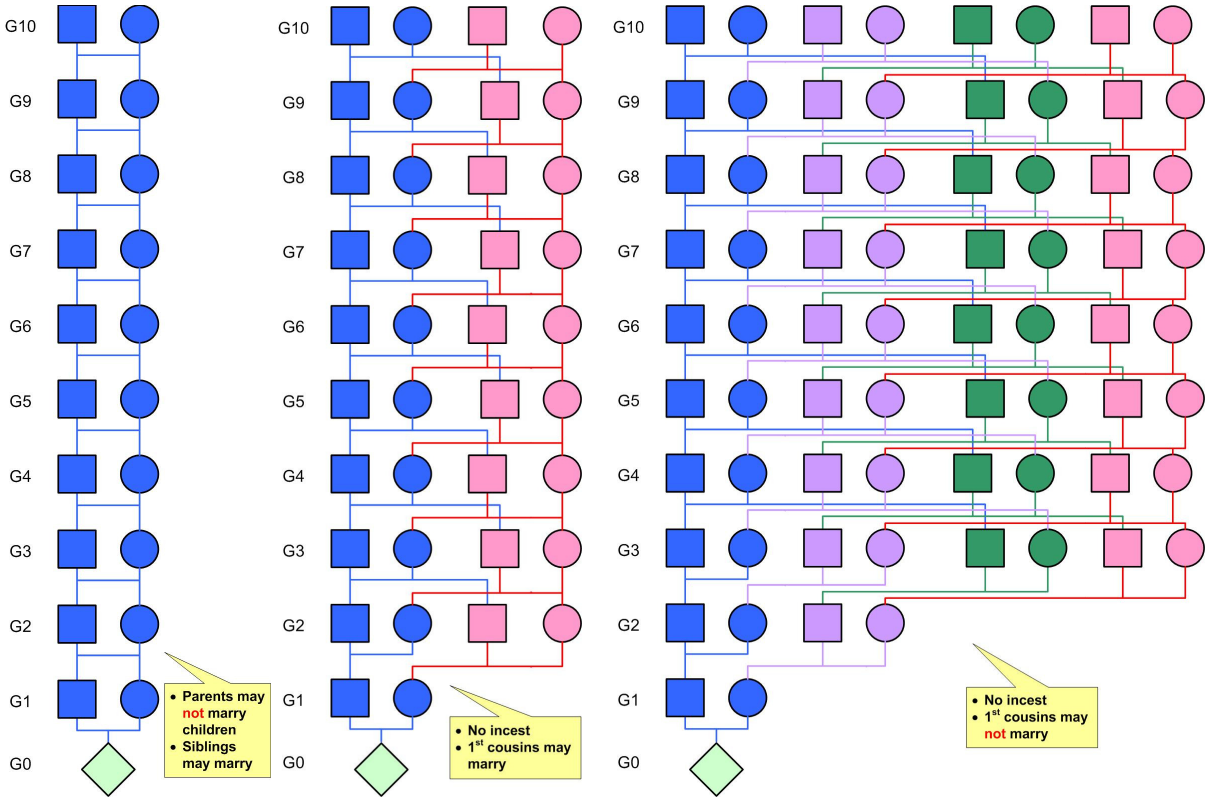

Figure 1 3 Marriage Cases: Between Siblings, Between 1st Cousins, Between 2nd Cousins

Question 2: As we showed in the answer to Question 1, the minimal number of ancestors at generation 10 where 1st cousins can marry is 4. Note that in this minimal case each generation consists of 1st cousins.

Question 3: If we disallow marriages between 1st cousins (and disallow incest), then the third pattern on the right of Figure 1 shows that the minimal number of ancestors in generation 10 would be 8. Again note that in this minimal case each generation consists of 2nd cousins.

It should be clear that the minimal number of ancestors depends on the marriage restrictions between cousins of a given degree. For example, it seems reasonable to expect that if marriage was disallowed between siblings, 1st, and 2nd cousins, then the minimum number of ancestors in generation 10 would be 16. In other words, if cousins of degree n are allowed to marry, but not cousins at a lesser degree, then the minimal number of ancestors would be 2^{n+1} and that number would hold for all older generations. Notice that each generation k, where $k \leq n$, would only have 2^k ancestors.

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Humanity's Family Tree

(http://andrewsullivan.thedailybeast.com/2012/08/humanitys-family-tree-.html)

The Bible claims Jesus was a descendant of King David. Dr. Yan Wong shows[1] that this is true ("all of Jesus's contemporaries" were as well), and repeats the genealogical experiment for today's larger population:

> How far do we have to go back to find the most recent common ancestor of all humans alive today? Again, estimates are remarkably short. Even taking account of distant isolation and local inbreeding, the quoted figures are 100 or so generations in the past: a mere 3,000 years ago. And one can, of course, project this model into the future, too. The maths tells us that in 3,000 years someone alive today will be the common ancestor of all humanity.

[1] http://www.bbc.co.uk/news/magazine-19331938