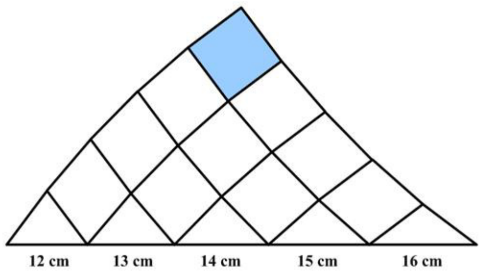
Stacked Rhombuses Puzzle

25 August 2025

Jim Stevenson



This is puzzle[1] from Talwalkar's set of "6 Impossible Puzzles with Surprising Solutions."

Call this puzzle the leaning tower of rhombi.

There are 5 isosceles triangles, aligned along their bases, with base lengths of 12, 13, 14, 15, 16 cm. The 10 quadrilaterals above are in rows of 4, 3, 2, and 1. Each quadrilateral is a rhombus, and the top of the tower is a square. What is the area of the square?

My Solution

For me this certainly qualifies as a Coffin Problem.[2] I spent almost two weeks trying to solve it. I will give my solution and then indicate some of my dead ends.

Since the sides *s* of the rhombuses are parallel (and equal), we can extend the sides of the outer stacks of rhombuses to form a grid of orthogonal, parallel lines, starting with the blue square (Figure 1). We see that this means the right side of the base 12 cm triangle is perpendicular to the left side of the base 16 cm triangle. From this we conclude we have two congruent 3-4-5 right triangles (Figure 2), which implies the side *s* of the square is 10 cm. So the area is 100 cm^2.

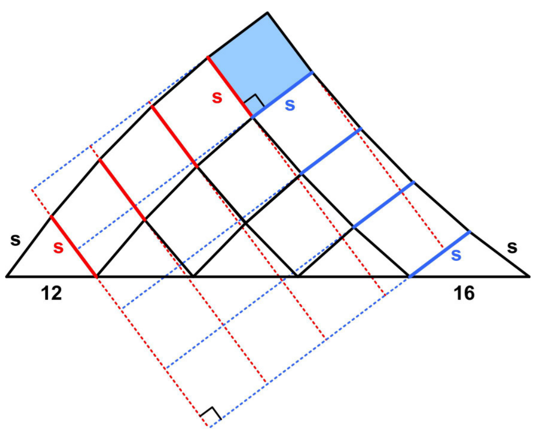


Figure 1

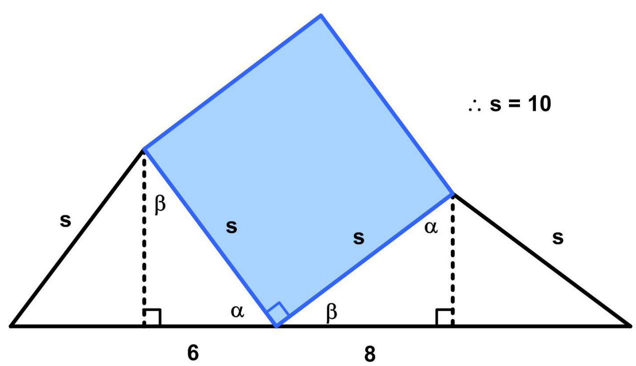


Figure 2

Dead Ends

I thought at first that the horizontal diagonals of the rhombuses might lie on straight lines that converged to an external point (Figure 3). Then I would proceed as follow:

1. Prove the diagonals of the rhombuses lie on

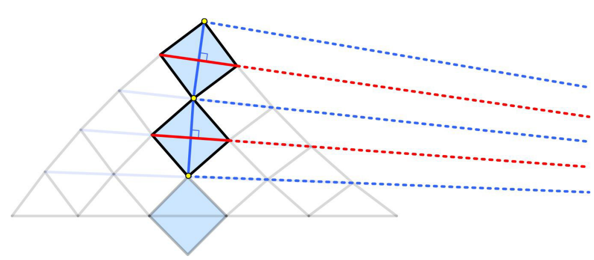


Figure 3

[1] 10 August 2025 (https://mindyourdecisions.com/blog/2025/08/10/6-impossible-puzzles-with-surprising-solutions-2/)

[2] "Three Coffin Problems" (https://josmfs.net/2019/01/19/three-coffin-problems/)

straight lines.

2. Prove these lines extended all intersect at a common external point.
3. Prove the angles the lines make are all equal.
4. Prove the blue square will be congruent to the middle, 14 cm rhombus.

It took a lot of calculating and Visio drawings to see the assumption that the diagonals of the rhombuses lay on straight lines was false (Figure 4). So that approach was bogus.

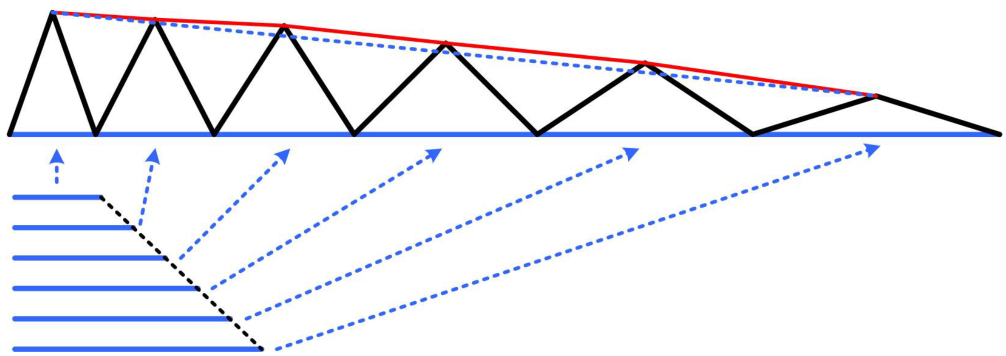


Figure 4 Triangle bases increase by a constant increment. Top vertices are not collinear.

Then I tried computing relationships between all the angles in the rhombuses (Figure 5). This was a nightmare, but it revealed that the rhombuses in the left stack seemed to rotate clockwise in equal increments. Similarly, the rhombuses in the right stack seemed also to rotate clockwise in equal increments.

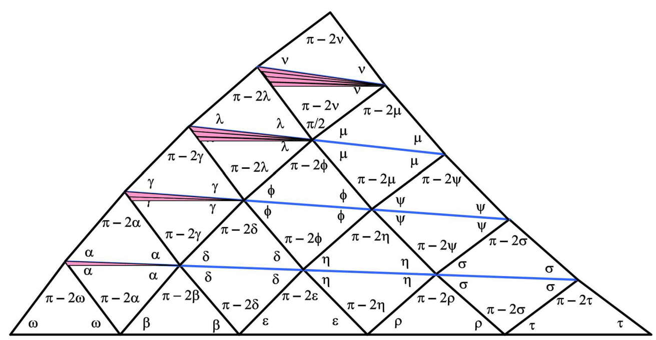
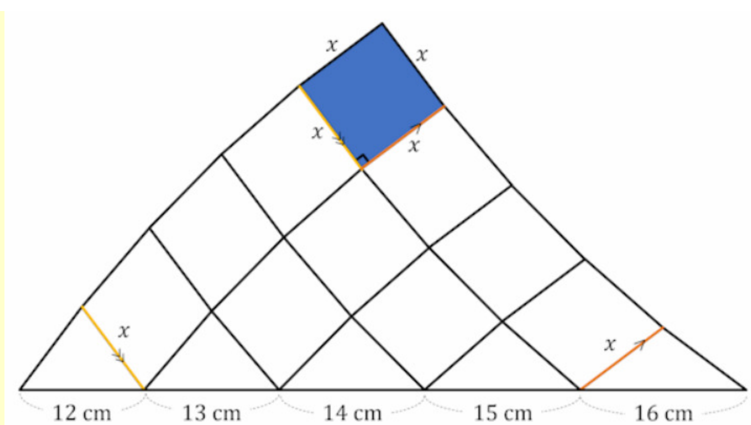


Figure 5

That made me think about the relationship between the right side of the base 12 triangle and left side of the base 16 triangle. So I extended their sides so that they would intersect and found they seemed to be perpendicular (Figure 1). And thus the final solution was born.

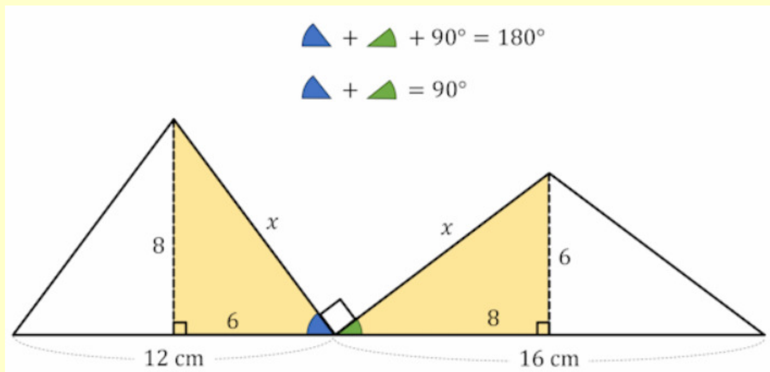
Talwalkar's Solution

This is the same solution as the one I finally found.



Let the square's side length be *x*. The bottom left side of the square will be parallel to every bottom left side of a rhombus, which goes all the way down to the right side of the 12 cm base triangle. This must have the same length x since all sides of a rhombus are equal. Similarly, the bottom right side of the square will be parallel and have the same length as the left side of the 16 cm base triangle.

The yellow and orange sides are perpendicular in the square, so they must be orthogonal on the base. We can vanish all the edges and just place those 2 triangles next to each other. Drop altitudes on those triangles, dividing the bases into half, and creating 2 right triangles. The two acute angles and the 90 degree angle form a line, so the two acute angles are complementary. So we have similar right triangles with congruent hypotenuses, meaning these two right triangles are congruent. The altitudes of each triangle are the half-base of the other.



We thus have a right triangle with legs of 6 and 8, so the hypotenuse is:

$$x^2 = 6^2 + 8^2$$

$$x^2 = 100$$

The square's side is x, so its area is precisely $x^2 = 100$ cm. And that's it!

References

https://www.sineofthetimes.org/a-geometry-challenge-from-japan/

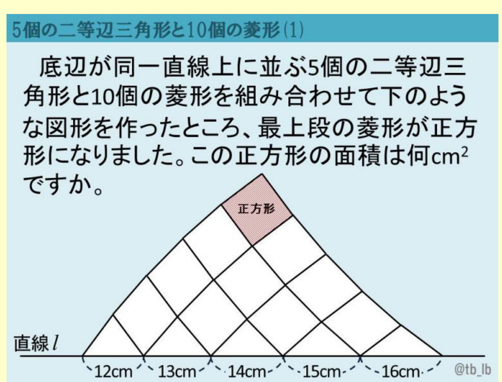
https://pororocca.com/problem/1101/

I decided to look at Talwalkar's first reference. I found Scher's discovery approach instructive.

A Geometry Challenge from Japan

Daniel Scher, March 14, 2017

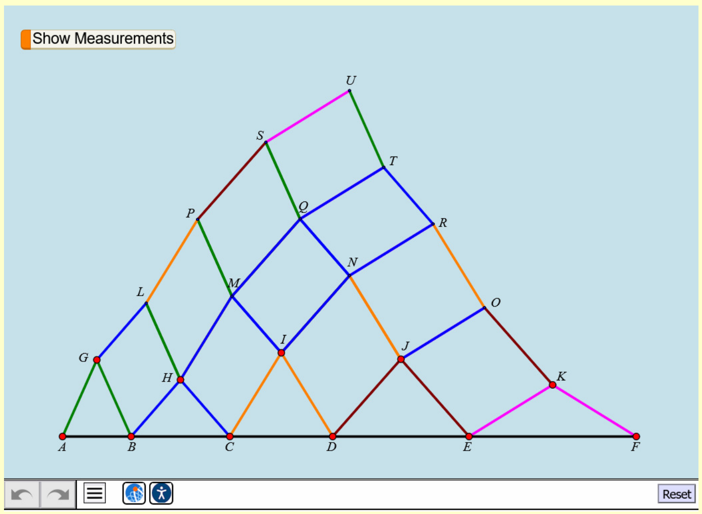
Here is a wonderful geometry problem from Japan: The five triangles below are all isosceles. The quadrilaterals are all rhombi. The shaded quadrilateral is a square. What is the area of the square?



I wondered at first whether the English translation of the problem was correct because with so many side lengths unspecified, it was hard to believe that the square's area could be uniquely determined. But the sheer strangeness of the problem led me to persist. And as with many geometry problems, I turned to Sketchpad to look for insights.

Building the isosceles triangles with the specified base lengths seemed laborious so I made the lengths adjustable. I also made one other significant change: rather than constructing rhombi (which led to a rather boring interactive model), I built parallelograms instead. This made for more interesting interactivity and did not change the essential elements of the solution.

With so many isosceles triangles and parallelograms in the construction, I decided to color-code the segments to make it easier to identify those that were equal in length. Below … is a Web Sketchpad version of my work. Before reading further, try dragging points *G, H, I, J,* and *K* to make your own discoveries.



As I dragged the points, I looked for invariants—those elements of the construction that remained the same while others changed. I was initially surprised that dragging point *H* translated quadrilaterals *SQTU, PMQS,* and *LHMP* while keeping their side lengths and angles intact But as I thought about it, the behavior made sense once I accounted for all the dependencies in the construction. I can imagine a great class discussion that focuses on explaining what changes and does not change—and why—as each point is dragged.

One observation particularly relevant to the problem was that for *SQTU* to be a square, *GB* and *KE* needed to be of equal length. I also noted that for $\angle SQT$ to be 90°, *GB* must be perpendicular to *KE* since these two segments are parallel to *SQ* and *TQ,* respectively. To view the length and angle measurements in the websketch above, press the *Show Measurements* button. You can then make *SQTU* a square by adjusting points *B* and *G*.

Given that the state of *SQTU* depended entirely on Δ*ABG* and Δ*EFK*, I realized that I could collapse the construction into one that showed only the essential elements needed to solve the problem. In order, I dragged point *C* to *B,* point *D* to *C,* and point *E* to *D,* leaving the construction shown below. With all the extraneous elements of the construction gone, you should now be able to determine the area of the square.