Curious Calendar Puzzle

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This is a curious relation from the 2024 Math Calendar ([1]).

10! seconds is exactly how many weeks?

As before, recall that all the answers are integer days of the month, and also recall that 10! = 10×9×8 … ×3×2×1.

Solution

So we want to find x weeks, such that

10! sec = x weeks × 7 days/week × 24 hr/day × 60 min/hr × 60 sec/min

So factor $7 \times 24 \times 60 \times 60$ into prime factors:

$$7 \times 24 \times 60 \times 60 = 7 \times 5^2 \times 3^3 \times 2^7$$

$$= \mathbf{10} \times 7 \times \mathbf{5^1} \times 3^3 \times \mathbf{2^6}$$

$$= 10 \times \mathbf{9} \times 7 \times 5 \times \mathbf{3^1} \times 2^6$$

$$= 10 \times 9 \times \mathbf{8} \times 7 \times 5 \times 3^1 \times \mathbf{2^3}$$

$$= 10 \times 9 \times 8 \times 7 \times \mathbf{6} \times 5 \times \mathbf{3^0} \times \mathbf{2^2}$$

$$= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \mathbf{4}$$

Then $$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times x$$

implies $$x = 3 \times 2 = 6 \text{ weeks}$$

It is surprising that the number of weeks came out evenly.

References

[1] Rapoport, Rebecca and Dean Chung, *Mathematics 2024: Your Daily epsilon of Math*, American Mathematical Society, 2024. August