Impossible Homework

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This is a somewhat unusual problem[1] from Presh Talwalkar. It involves proving a student's homework problem is impossible.

I came across a homework problem described as "scary" on Reddit AskMath. You need to fill in the number sentences using the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 exactly once.

You should try a few possibilities to see why this is a challenging question. And do not waste too much time because the exercise is literally impossible! The challenge is, can you prove no solution exists?

Solution

I admit I was at a loss as to how to proceed. It reminded me of the many stupid "problems" that involve "order of operations" in ambiguous settings devoid of clarifying parentheses that no legitimate mathematician would ever write. So I was going to ignore it, until I read the solution. Then I realized it was a great example of the power of symbolic algebra, which is basically doing arithmetic with symbols. So I will give Talwalkar's solution.

Thanks Don for alerting me of a typo!

This solution is by u/FilDaFunk.[2]

Suppose the problem can be solved, and let the boxes be filled with unknown constants $a, b, c, d, e, f, g, h, i, j$. The number sentences are then the equations:

$$a + b - c = d$$
$$e - f = g$$
$$h + i = j$$

Convert all equations so they only have additions. Add a c to both sides of the first equation, and add f to both sides of the second equation.

$$a + b = c + d$$
$$e = f + g$$
$$h + i = j$$

Now add all three equations together.

$$a + b + e + h + i = c + d + f + g + j$$

Let us sum all 10 of the unknown constants.

$$a + b + c + d + e + f + g + h + i + j = (a + b + e + h + i) + (c + d + f + g + j)$$
$$= (a + b + e + h + i) + (a + b + e + h + i)$$
$$= 2(a + b + e + h + i)$$

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1 7 February 2025 (https://mindyourdecisions.com/blog/2025/02/07/scary-homework-is-impossible/)
2 https://www.reddit.com/r/askmath/comments/1i5yy92/comment/m87uwms/

But we can sum the 10 unknown constants in another way. The 10 numbers together are the whole numbers from 0 to 9, in some order, so we have:

$$a + b + c + d + e + f + g + f + g + j = 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$$

$$= 9(10)/2$$

$$= 45$$

We have summed the unknown constants in two different ways, giving:

$$45 = 2(a + b + e + h + i) \tag{*}$$

$$45/2 = a + b + e + h + i$$

But each unknown constant is an integer, and the sum of integers is an integer. However if a solution exists, the sum of these 5 integers will be a non-integer $45/2 = 22.5$.[3] This is not possible! Therefore the assumption a solution exists is wrong, and we can conclude there is no solution to this exercise!

Students often complain their math homework is impossible. But this is a case where the problem is literally impossible!

Reference

Reddit AskMath:

https://www.reddit.com/r/askmath/comments/1i5yy92/the_math_book_of_my_cousin_is_scary/

Euphoric Comment. How cool is that? Somehow codifying the unknowns of the problem and manipulating them according to the patterns and rules of arithmetic eventually leads to a solution. What a marvel of the human mind that took literally thousands of years to achieve. The metaphor of mathematics as a cathedral of the mind is perfect. It took many human beings years to build and it reflects a power that reaches to the heavens. Learning enough to appreciate this endeavor is a gift. Yes, mathematics is a useful tool and humans are above all great tool makers, but that is ultimately a by-product and bonus. Its kinship with other explorers of the imagination, such as music, art, and architecture, gives it another dimension. We should be grateful if we can experience even a small part of this masterpiece.

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[3] JOS: Or we could argue directly, staying in the realm of whole numbers, that equation (*) implies 2 is a factor of 45, which is false. (Which is another way of saying equation (*) implies 45 is even when it is in fact odd.)