

Tire Wear

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Thanks to Futility Closet I discovered a new source of math puzzles: A+Click.¹

A+ Click helps students become problem solvers. Free, without ads, no calculators, and without signing-up. The website features a graduated set of 16,000+ challenging problems for students in grades one through twelve, starting from the very simple to the extremely difficult. ... The questions concentrate on understanding, spatial reasoning, usefulness, and problem solving rather than math rules and theorems. The problems include a short description and an illustration to help problem solvers visualize the model. The problems can be solved within one minute and without using a calculator.

My only quibble with “The questions concentrate on understanding, spatial reasoning, usefulness, and problem solving rather than math rules and theorems.” is that by keeping explicit math notation and concepts to a minimum, the use of symbolic algebra and calculus is muted and there is a whiff of the medieval reliance on mental verbal agility rather than the power of the new mathematics.

Still the problems are imaginative and challenging. Here is a good example.²

The rear tires of my car wear out after 40,000 miles, while the front tires are done after 20,000 miles. Estimate how many miles I should drive before the tires (front and rear) are rotated to drive the maximal distance.

Answer Choices: 15,000 miles 12,000 miles 13,333 miles 16,667 miles

(I admit solving these under a minute is a challenge, at which I often failed. Ignoring time constraints allows for greater care and a more thorough mulling over the intricacies of the problem. Yes, those who have mastered math can solve problems faster than those who have not, but real mastery of math requires an inordinate attention to details, and that requires time.)

My Solution

Assume the life of a tire is 40 thousand miles. Let x be the distance driven before rotation and y the distance after rotation, and let d be the total distance, namely, $d = x + y$. By symmetry we only need to consider one back tire and one front tire. Let r_1 and r_2 be the life remaining in the two tires, respectively, at any moment, and let r be the sum of the amount of tire life left in the tires. So initially $r = r_1 + r_2 = 40 + 40 = 80$. Then after the car has been driven x miles we have

$$r = \text{back life} + \text{front life} = r_1 + r_2 = (40 - x) + (40 - 2x)$$

since the wear on the front tire is twice that of the back tire. After rotation and when the car has been driven y miles, we have

$$r = \text{back life} + \text{front life} = r_2 + r_1 = (40 - 2x) - y + (40 - x) - 2y = 80 - 3(x + y) = 80 - 3d$$

The maximum distance d we can drive is when we leave no remaining life on the tires, that is, when $r = 0$. Therefore the distance will be $d = 80/3$. Since $r_1 \geq 0$ and $r_2 \geq 0$, $r = 0 \Rightarrow r_1 = r_2 = 0$. Now $r_1 = r_2 \Rightarrow x = y$. Therefore, $x = d/2 = 40/3 = 13.333$ thousand miles or **13,333 miles**.

¹ <https://aplusclick.org/index.html>

² <https://aplusclick.org/t.htm?level=11;t=2;q=4060>

Comment. It is interesting that if we don't rotate the tires, we can only drive 20,000 miles, since the front tires will wear out by then. But if we do rotate, even though the life of each tire is 40,000 miles, we can only drive 26,667 miles before they all wear out.

A+Click Solution

If X is the number of miles in thousands before rotating the tires, then the rear tires have used $X/40$ of their capacity and the front tires $X/20$. After the change, the old rear tires live $20(1 - X/40)$ thousand miles and the old front tires $40(1 - X/20)$. The distances are equal.

$$20(1 - X/40) = 40(1 - X/20)$$

$$0.075 X = 1$$

$$X = 13.333 \text{ thousand miles}$$

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