

Dan Steinitz Solution

29 January 2025

Hello,

I stumbled upon your great blog site (after following a thread about Catriona Agg.) As a math teacher I ought to thank you for this unique contribution. I have a comment about a specific problem you address: the tired messenger. <https://josmfs.net/2023/04/22/the-tired-messenger-problem/>

I was already acquainted with the problem as an example for the power of shifting frames of reference: If you solve this problem from the point of view of the train driver [cyclist], you immediately notice that the (relative) velocity of the messenger is a vector with a given direction (with a X:Y ratio given by the initial X and Y distances between driver and messenger.)

Now, [Figure 1] this relative velocity $[V_B]$ is the velocity of the messenger (in the ground frame) $[V'_B]$ minus the train [cyclist] velocity (ground frame) $[U]$, or

$$-U + V \quad [V_B = V'_B - U]$$

if $-U$ is minus the train [cyclist] velocity, V the messenger velocity

The resulting vector has to lie on the required direction, hence the minimal V is the vector that is perpendicular to the required direction. This yields the solution immediately [Figure 2].

It is amusing to see the “mathematicians” solutions compared to the elegance and simplicity (and rigour) of this solution.

Sincerely grateful,

Dan Steinitz

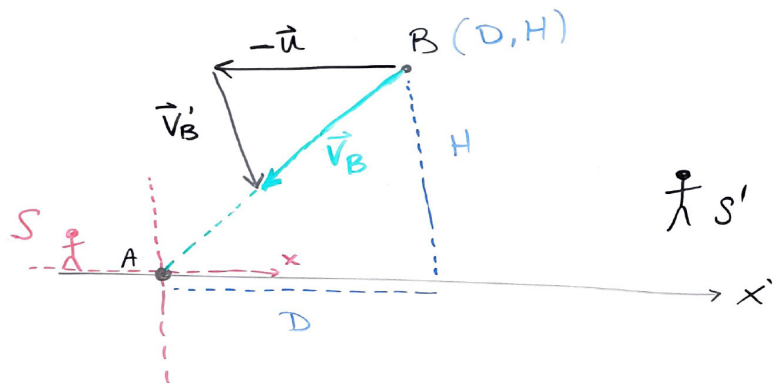


Figure 1

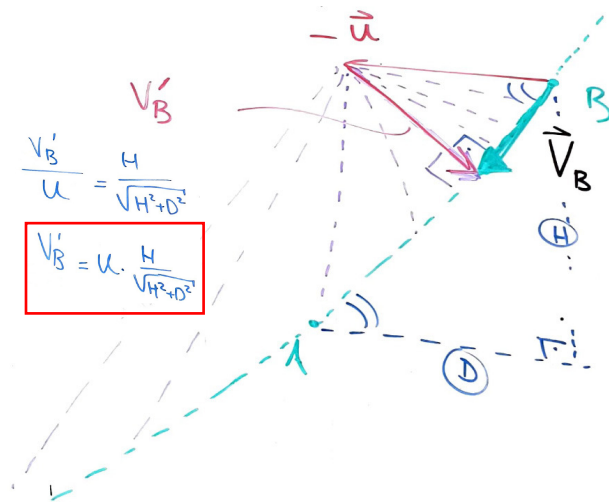


Figure 2

$$\frac{V'_B}{U} = \frac{H}{\sqrt{H^2 + D^2}}$$

$$V'_B = U \cdot \frac{H}{\sqrt{H^2 + D^2}}$$