NSA Track and Field Puzzle

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This is a puzzle from Futility Closet.[1]

A puzzle by Steven T., a systems engineer at the National Security Agency, from the NSA's September 2016[2] Puzzle Periodical[3]:

Three athletes (and only three athletes) participate in a series of track and field events. Points are awarded for 1st, 2nd, and 3rd place in each event (the same points for each event, i.e. 1st always gets "x" points, 2nd always gets "y" points, 3rd always gets "z" points), with $x > y > z > 0$, and all point values being integers.

The athletes are named Adam, Bob, and Charlie.

- Adam finished first overall with 22 points.
- Bob won the Javelin event and finished with 9 points overall.
- Charlie also finished with 9 points overall.

Question: Who finished second in the 100-meter dash (and why)?

I thought this puzzle impossible at first. There didn't seem to be enough information to solve it. But a bit of trial-and-error opened a path.

My Solution

This is more of a puzzle in logic than math. The first thing to notice is that we don't know how many events there are or what they are. Apparently we have to figure that out as part of the solution.

As a first cut, consider Table 1. We have N events where $N \geq 2$ and Bob wins the javelin contest. We will assume Adam wins the rest and comes in second in the javelin. That is plausible, since he is a better athlete than Charlie. That makes Charlie come in third in the javelin. Since we don't know what the other events are besides the 100 meter, we will assume they all have the same distribution of scores. Since Bob's javelin score is higher than Charlie's, Charlie's later scores must be higher than Bob's. So we get the initial pattern as shown in the table.

Table 1

Events	Javelin	100 Meter	3	4	5	…	N	Points	Points
Adam	y	x	x	x	x			22	(N - 1)x + y
Bob	x	z	z	z	z			9	N x + (N - 1)z
Charlie	z	y	y	y	y			9	z + (N - 1)y

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[1] https://www.futilitycloset.com/2024/04/29/javelin/

[2] https://www.nsa.gov/Press-Room/News-Highlights/Article/Article/1627324/september-puzzle-periodical-javelin/

[3] https://www.nsa.gov/Press-Room/News-Highlights/Tag/120382/puzzle/

Now this is a bit difficult to analyze, so suppose we write things in terms of the least score z. That is, assume there are whole numbers n > m so that x = z + n and y = z + m (so x > y > z > 0). Then Table 1 becomes Table 2.

Table 2

Events	Javelin	100 Meter	3	4	5	…	N	Points	Points
Adam	z + m	z + n	z + n	z + n	z + n		z + n	22	Nz + m + (N-1)n
Bob	z + n	z	z	z	z		z	9	Nz + n
Charlie	z	z + m	z + m	z + m	z + m		z + m	9	Nz + (N – 1)m

Now lets assume first that **z = 1**. Then we get the possibilities for the competitors' distribution of points for the number of events N ranging from 2 to 6 as shown in Table 3.

Table 3

N	2	3	4	5	6	Points
Adam	2 + m + n	3 + m +2 n	4 + + m + 3n	5 + m + 4n	6 + m + 5n	22
Bob	2 + n	3 + n	4 + n	5 + n	6 + n	9
Charlie	2 + m	3 + 2m	4 + 3m	5 + 4m	6 + 5m	9

Case N = 2

This implies n = m, since Bob and Charlie have the same total points. But that contradicts n > m. So the number of games N > 2.

Case N = 3

This implies n = 2m, Therefore from Charlie we have

$$9 = 3 + 2m \Rightarrow m = 3 \text{ and } n = 6.$$

But for Adam this means

$$22 = 3 + m + 2n = 6 + 12 = 18,$$

a contradiction. Therefore N > 3.

Case N = 4

This implies n = 3m and so from Charlie

$$9 = 4 + 3m \Rightarrow m = 5/3,$$

which contradicts the fact the numbers are integers.

Case N = 5

This implies n = 4m and so from Charlie

$$9 = 5 + 4m \Rightarrow m = 1 \text{ and } n = 4.$$

This in turn implies from Adam that

$$22 = 5 + m + 4n = 6 + 16 = 22 \checkmark$$

So there are 5 events and the points are x = 5, y = 2, and z = 1. Charlie comes in second in all but the javelin. So in particular, he comes in second in the 100 meter race.

Technically, I haven't proved there isn't another solution. Probably by considering the impossibility of Case N = 6, and then the cases when z > 1 we could prove uniqueness. We would also have to show the assumption that Adam had z as a score for the javelin was impossible.

The Futility Closet solution solves the uniqueness question by noticing the total number of points awarded 40 has to equal the number of events N times the same sum of x + y + z for each event, namely,

$$N (x + y + z) = 22 + 9 + 9 = 40$$

Therefore N divides 40 and that eliminates a lot of cases, as they show. And the fact that x + y + z also divides 40 and satisfies x > y > z > 0 further limits the cases—in fact, to one.

Futility Closet (NSA) Solution

First, realize that the total points awarded is 40. Given that we are dealing with integers that means the total number of Events (TE) times the total number of points awarded in an event (TP) must equal 40. We can quickly eliminate some possibilities.

Events (TE)	Points (TP)	
1	40	Not possible since we know there are at least two events.
2	20	Not possible since Adam and Bob both won 1 event. There would be no way for Bob and Charlie to finish with the same points.
4	10	A possibility.
5	8	A possibility.
8	5	Not possible, since 1st > 2nd > 3rd > 0 means the minimum number of points awarded HAS to be 6 (3, 2, 1).
10	4	Same reason.
20	2	Same reason.
40	1	Same reason.

So we now know it was either 4 events with 10 points awarded in an event or 5 events with 8 points awarded. Let's look at the 4/10 situation first. The possible points for each place with 10 overall points available is (all other combinations are impossible because of the 1st > 2nd > 3rd > 0 constraint):

- 5, 3, 2 Not possible as there is no way to get to 22 points in 4 events (greatest possible points would be 20).
- 5, 4, 1 Same reason.
- 6, 3, 1 Not possible, since no combination of 4 numbers can get to 22 points (3 × 6 + 3 = 21, 4 × 6 = 24).
- 7, 2, 1 A possibility — let's look further.

Can we get Adam to 22? Yes. Adam can finish 1st 3 times and 3rd once (in the Javelin). 3 × 7 + 1 = 22.

Can we get Bob to 9? Nope. Bob won the Javelin (7 points). There's no way to get to 9 with the remaining event/point combinations. Sooooo …

There must be 5 events with a total of eight points (we're getting close). Let's look at the possible points (again, the other combinations are not possible because of the 1st > 2nd > 3rd > 0 constraint):

- 4, 3, 1 Not possible as there is no way to get to 22 points (4 × 5 = 20 is maximum)
- 5, 2, 1 Looks like this is the winner. Let's check.

Can we get Adam to 22? Yes. Adam can finish 1st 4 times and 2nd once (4 × 5 + 2). Since we know Bob finished 1st in the Javelin, Adam must have finished 2nd.

Can we get Bob to 9? Yes. Bob finished 1st in the Javelin (5 points) and finished 3rd 4 times (4 ×1) for a total of 9 points.

Can we get Charlie to 9? Yes. Charlie finished 3rd in the Javelin (1 point) and must have finished 2nd in the 4 other events (4 × 2) for a total of 9 points.

Since Charlie finished 2nd in EVERY EVENT OTHER THAN THE JAVELIN, he MUST have finished 2nd in the 100-meter dash. Here is a little table:

	Javelin	Event 2	Event 3	Event 4	100m Dash	Total
Adam	2nd (2 pts)	1st (5 pts)	1st (5 pts)	1st (5 pts)	1st (5 pts)	22 pts
Bob	1st (5 pts)	3rd (1 pt)	3rd (1 pt)	3rd (1 pt)	3rd (1 pt)	9 pts
Charlie	3rd (1 pt)	2nd (2 pts)	2nd (2 pts)	2nd (2 pts)	2nd (2 pts)	9 pts