Young and Old Problem

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This is a 2017 Center of Mathematics Problem of the Week ([1]).

The average age of a 10 member committee is the same as it was 4 years ago, because an older aged member has been replaced by a younger aged member. How much younger is the young member than the older member they replaced?

My Solution

Let a_1, a_2, a_3, … a_{10} be the current ages of the original committee with a_{10} being the age of the older member being replaced. Let y be the current age of the young replacement. Then we have

$$\frac{\sum_{k=1}^{10}(a_k - 4)}{10} = \frac{\sum_{k=1}^{9}a_k + y}{10}$$

so

$$\sum_{k=1}^{9}a_k + a_{10} - 40 = \sum_{k=1}^{9}a_k + y$$

or

$$a_{10} - 40 = y$$

So the young replacement is 40 years younger than the older original member.

Center of Math Solution

I find this solution a bit ambiguous in that the time of the ages is not clearly specified initially, since the problem statement indicates the ages are all current.

First, we can define some variables for the different ages of members in the committee for this problem.

The sum of the ages of the other nine members = x

The age of the younger member = y

The age of the older member = z

We can now write equations for the average four years ago and this year.

The average age four years ago:

$$(x + z) / 10$$

The average age this year: Because four years have passed we must account for the nine other members and the young member aging four years.

$$[x + y + (4 \times 10)] / 10$$

We can now write an equation using both averages.

$$(x + z) / 10 = [x + y + (4 \times 10)] / 10$$

$$z = 40 + y$$

We can see that the young member is 40 years younger than the older member they replaced.

References

[1] "Problem of the Week", *Center of Mathematics*, November 16, 2017. (https://centerofmathematics.blogspot.com/2017/11/think-thursday-committee-algebra.html)

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