Oscar Rojas Solution

Here is my approach:

$$\frac{a^3 - b^3}{(a-b)^3} = \frac{(a-b)(a^2 + ab + b^3)}{(a-b)(a^2 - 2ab + b^3)} = \frac{a^2 + ab + b^3}{a^2 - 2ab + b^3} = \frac{73}{3}$$

After some simplification:

$$70(a^2 + b^2) = 149ab$$

149 is prime, so it divides $a^2 + b^2$. Therefore $(a^2 + b^2) = 149k$ for some integer k, and then $ab = 70k$.

Now $\alpha = a^2$ and $\beta = b^2$ can be seen as roots of a quadratic equation in x via

$$(x - \alpha)(x - \beta) = x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (a^2 + b^2)x + a^2b^2 = x^2 - 149k\,x + 4900k^2 = 0$$

Using the quadratic formula yields

$$x = (149k \pm 51k)/2 = 100k \text{ or } 49k$$

So, since $a > b$,

$$\alpha = a^2 = 10^2k \text{ and } \beta = b^2 = 7^2k$$

Since 7 is prime, 7 must divide b. So there is an integer c such that $b = 7c$. This means $k = c^2$. Thus

$$a = 10c \text{ and } b = 7c$$

But a and b are relatively prime, so $c = 1$, and $a = 10$, $b = 7$, and $a - b = 3$.