

Air Travel

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Jim Stevenson

This is a nice problem from *Five Hundred Mathematical Challenges* ([1]).



Problem 62. A plane flies from A to B and back again with a constant engine speed. Turn-around time may be neglected. Will the travel time be more with a wind of constant speed blowing in the direction from A to B than in still air? (Does your intuition agree?)

Solution

Let the plane speed be v and the wind speed $w < v$. Let the distance between A and B be D . Let t be the round-trip time for the plane without wind, t_1 the time from A to B with the wind, and t_2 the time from B to A against the wind. Then we have the following equations.

$$2D = vt$$

$$D = (v + w)t_1$$

$$D = (v - w)t_2$$

Therefore

$$t_1 + t_2 = \frac{D}{v + w} + \frac{D}{v - w} = v \frac{t}{2} \left(\frac{2v}{v^2 - w^2} \right) = t \frac{1}{1 - \left(\frac{w}{v}\right)^2}$$

So for $0 < w < v$, $t_1 + t_2 > t$, that is, the total round-trip travel time with the wind is greater than it would be without the wind.

References

- [1] Barbeau, Edward J., Murray S. Klamkin, William O. J. Moser, *Five Hundred Mathematical Challenges*, Spectrum Series, Mathematical Association of America, Washington D.C, 1995

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