Three Dutchmen Puzzle

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Jim Stevenson


Dudeny 1917

Presh Talwalkar presented an interesting puzzle ([1]) that originated in the *Ladies' Diary* of 1739-40 ([2]), was recast by Henry Dudeney in 1917 ([3]), and further modified using American money.

Each of three Dutchmen, named Hendrick, Elas, and Cornelius has a wife. The three wives have names Gurtrün, Katrün, and Anna (but not necessarily matching the husband's names in that order). All six go to the market to buy hogs.

Each person buys as many hogs as he or she pays dollars for one. (1 hog costs \$1, 2 hogs are \$2 each, 3 hogs cost \$3 each, etc.) In the end, each husband has spent \$63 more than his wife. Hendrick buys 23 more hogs than Katrün, and Elas 11 more than Gurtrün. Now, what is the name of each man's wife?

Solution

Let h be the number of the hogs the husband bought and w the number the wife bought. Then the cost for the husband is h hogs × h dollars/hog and for the wife w hogs × w dollars/hog. We are given that for all the husbands and wives,

$$h^2 - w^2 = 63$$

So

$$(h + w)(h - w) = 63 \cdot 1 = 21 \cdot 3 = 9 \cdot 7$$

which provides the three possibilities for the number of hogs bought by the three couples. Solving the three cases yields:

	63·1	**21·3**	**9·7**
h + w	63	21	9
h – w	1	3	7
h	(63 + 1)/2 = 32	(21 + 3)/2 = 12	(9 + 7)/2 = 8
w	(63 – 1)/2 = 31	(21 – 3)/2 = 9	(9 – 7)/2 = 1

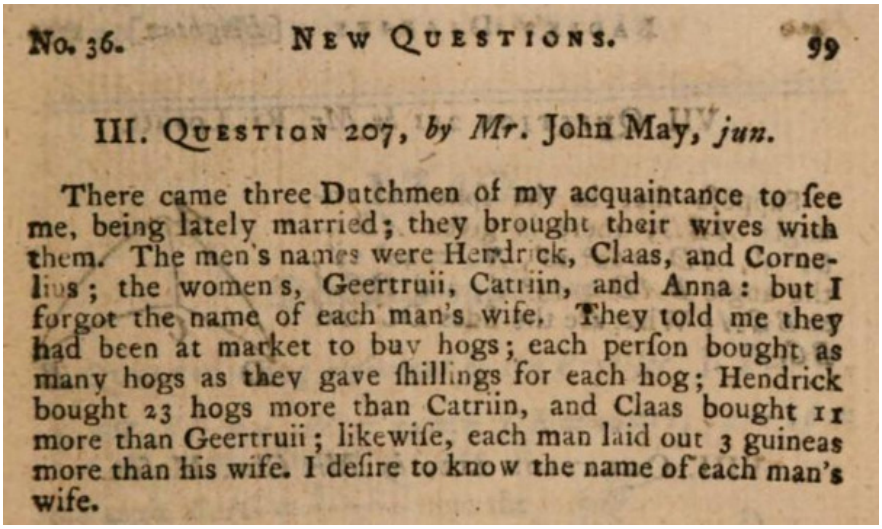
Now 32 – 9 = 23 and 12 – 1 = 11, and no other differences between husbands and other wives equal the ones stated in the problem. So that means Hendrick has 32 hogs, Katrün 9 hogs, Elas 12 hogs, and Gurtrün 1 hog. Therefore, the third husband Cornelius must have the 8 hogs, and the third wife Anna must have the 31 hogs.

And so we have the following pairing of wives to husbands:

	63·1	**21·3**	**9·7**
h	32 Hendrick	12 Elas	8 Cornelius
w	31 Anna	9 Katrün	1 Gurtrün

Comment. The approach I took turns out to be the same that Talwalkar followed. But the original solutions are all a bit more complicated (see Appendix below p.3). The link to the original solutions given in *The Diarian Miscellany*, rather than directly to the *Ladies' Diary*, came from Venkat's post on the problem ([4]).

The original problem statement is ([2]):



No. 36. NEW QUESTIONS. 99

III. QUESTION 207, *by Mr.* John May, *jun.*

There came three Dutchmen of my acquaintance to see me, being lately married; they brought their wives with them. The men's names were Hendrick, Claas, and Cornelius; the womens, Geertruii, Catriin, and Anna: but I forgot the name of each man's wife. They told me they had been at market to buy hogs; each perſon bought as many hogs as they gave ſhillings for each hog; Hendrick bought 23 hogs more than Catriin, and Claas bought 11 more than Geertruii; likewiſe, each man laid out 3 guineas more than his wife. I deſire to know the name of each man's wife.

It should be noted that 1 guinea = 21 shillings in 1740. So the numbers come out the same where dollars have been substituted for shillings. Also note that in this case men, and not women, both posed and solved the problem, which became a trend in later issues of the *Ladies' Diary*, since the men had few similar publications.[1]

As usual, Dudeney's solution ([3] pp.167-8) at first more or less gives the answer without an explanation. Then he launches into a general discussion about writing numbers as the difference of two squares, which finally ends with the technique used by Talwalkar and me (and Venkat).

References

[1] Talwalkar, Presh, "Impossible Dutchmen's Wives Logic Puzzle", *Mind Your Decisions,* 29 August 2024. (https://mindyourdecisions.com/blog/2024/08/29/impossible-dutchmens-wives-logic-puzzle/)

[2] Hutton, Charles, *The Diarian Miscellany: Consisting of All the Useful and Entertaining Parts, Both Mathematical and Poetical, extracted from the Ladies' Diary, From the beginning of that work in the year 1704, down to the end of the year 1773, With many additional Solutions and Improvements, in Five Volumes*, Vol II., Robinson and Baldwin, London, 1775.
Problem: p.99 (https://books.google.com/books?id=sRo3AAAAMAAJ&pg=PA99)
Solutions: p.104 (https://books.google.com/books?id=sRo3AAAAMAAJ&pg=PA104)

[3] Dudeney, Henry Earnest, "The Dutchmen's Wives", *Amusements in Mathematics*, 1917. Problem 139, p.26

[4] Venkat, "Three Dutchmen", *Cantor's Paradise*, 27 September 2021 (https://www.cantorsparadise.com/three-dutchmen-f9f08ac19d73)

[1] See the "Ladies' Diary Problem" (https://josmfs.net/tag/ladies-diary/)

Appendix: The Diarian Miscellany - Ladies' Diaries Solutions ([2])

III. QUESTION 207 *answered by Mr.* J. Hill.

Call the number of hogs any woman bought x; the number her husband bought $x+n$; money laid out by the woman is xx shillings; money laid out by the husband is $xx+2nx+nn$ shillings. Equation $xx+2nx+nn=xx+63$. $\therefore x=\frac{63-n}{2n}$. If $n=1$, then $x=31$, and $x+n=32$; hence some woman bought 31 hogs, and her husband 32: If $n=3$, then $x=9$, and $x+n=12$; therefore some other woman bought 9, and her husband 12: If $n=7$, then $n+x=8$; $\therefore$ some woman bought 1, and her husband 8. Consequently

Hendrick bought 32,	and his wife Anna	31
Claas — — 12	— — — Catriin	9
Cornelius — 8	— — — Geertruii	1

Answered by Merones.

For the persons put	Men.			Women.		
	A,	B,	C,	P,	Q,	R,
Hogs	a,	e,	y,	$e-c$,	$a-b$,	u,
Money	aa,	ee,	yy,	$\overline{e-c}^2$,	$\overline{a-b}^2$,	uu.

Let $b=23$, $c=11$. Compare B with Q, then per question $ee-\overline{a-b}^2=63$ shillings; that is, putting $e=a+z$; $2az+zz+46a=592$; therefore $a=\frac{23-z}{2}+\frac{63}{2z+46}$; now 'tis evident the last term cannot be a whole number; therefore z in the first term must be an even number, so the last term $\frac{63}{2z+46}$ must be the half of a whole number; let $\frac{63}{z+23}=v$. Whence $z=\frac{63}{v}-23$; hence v must be either 1, 3, 7, 9, 21, or 63: From each of which is had $\left.\begin{matrix}a\ 54, 32, 14, 22, 24\\ e\ 32, 12, 12, 8, 8\end{matrix}\right\}$. And again comparing C with P, then $yy-ee+22e=184$; and we find $\left.\begin{matrix}y\ 12, 8, 8, 12, 32.\\ e\ 2, 10, 12, 20, 42.\end{matrix}\right\}$

Whence e must be the same in both suppositions; therefore 'tis 12, if the question be possible in whole numbers. But since the other two persons A, R, must be compared, therefore $aa-uu=63$: From hence $a=32$, $u=31$, $e=12$, and $y=8$; but comparing the men and women in any other manner, it will appear there is no other answer in whole numbers. Therefore Hendrick and Anna, Claas and Catriin, and Cornelius and Geertruii, are man and wife.

The same answered by Mr. Rob. Heath.

Let $x=$ the hogs bought by either Hendrick, Claas, or Cornelius; then xx will be the shillings they cost, and $xx-63$ the shillings their wives hogs cost, which (as whole hogs) must always be a square number; because the square root of the shillings laid out for each parcel is equal to the number of hogs. Let $x-y=$ the side of that square, then $xx-63=xx-2xy+yy$. Consequently, by reduction, $x=\frac{63+yy}{2y}$; whence we find y may be

Hogs.

$\left.\begin{matrix}1\\3\\7\end{matrix}\right\}$ Conseq. $x=\left\{\begin{matrix}32\\12\\8\end{matrix}\right\}$ bought by the men, coupled with $\left\{\begin{matrix}31\\9\\1\end{matrix}\right\}$ bought by their Wives. Whence are joined Hendrick and Anna, Claas and Catriin, Cornelius and Geertruii.

Mr. N. Farrer

Obſerves, that the number of hogs the three men and their reſpective wives bought will be expreſſed by three pair of numbers, the difference of whoſe ſquares muſt be 63. Now all the whole numbers whoſe ſquares will produce this difference are 1 and 8, 9 and 12, 31 and 32; therefore 8, 12, 32, the men bought; 1, 9, 31 the women.