Counting Tanks

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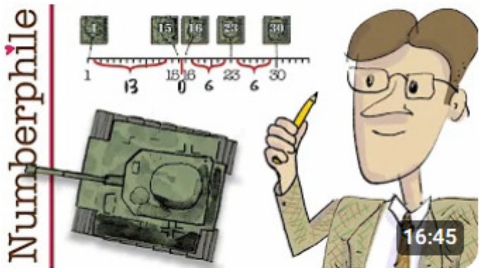
Jim Stevenson



A great example of the application of simple math to real-world problems is provided in a recent *Numberphile* video[1] on YouTube by the British mathematician James Grime ([1]).

It is taken from a real story from World War II where the British were trying to estimate the number of tanks the Germans were producing each month. The spies came up with an estimate of about 1500 tanks per month, whereas the mathematicians estimated the number to be closer to 250 tanks per month. How the mathematicians did this is explained by Grime. (I added an explanation of one step that whizzed by in Grime's presentation and that I didn't understand at first.)

Solution

The innovation of the mathematicians, contrary to the spies' attempt to count actual tanks, was to find out the serial numbers for various parts of a finished tank. Given such a collection for an observed sample of tanks the mathematicians would then estimate the total number of tanks that the sample could come from.

In the video Grime modeled the situation by holding a bag of toy tanks that had been numbered consecutively from 1 to some number N. He then selected 5 tanks from the bag and looked at the largest "serial number", 30, which he labeled N_{Max}. Then an estimate for the total number of tanks N should be bigger than then this maximum 30. Grime claimed the estimated number of tanks would be

$$N = N_{\text{Max}} + (N_{\text{Max}} - 5)/5 = 30 + 25/5 = 35,$$

or in general

$$N = N_{\text{Max}} + (N_{\text{Max}} - k)/k \qquad (*)$$

where k is the number of tanks sampled from the bag.

In order to show where this comes from, Grime laid the sample of serial numbers along a number line (Figure 1). (The actual first sample number was 1, but to explain the idea he considered the more likely case where the first number would be greater than 1.)

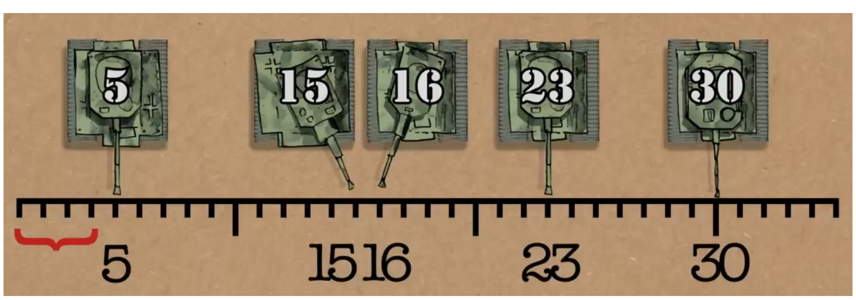


Figure 1

[1] https://www.youtube.com/watch?v=WLCwMRJBhuI

Then an estimate of how far above N_{Max} the actual total N would be is the gap in front of the first number. But that could vary wildly. So a better estimate might be the average of all the gaps (Figure 2 and Figure 3).

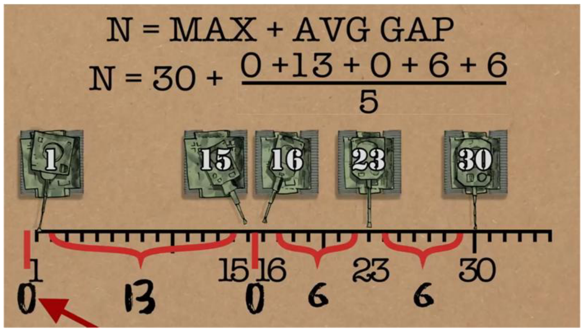


Figure 2

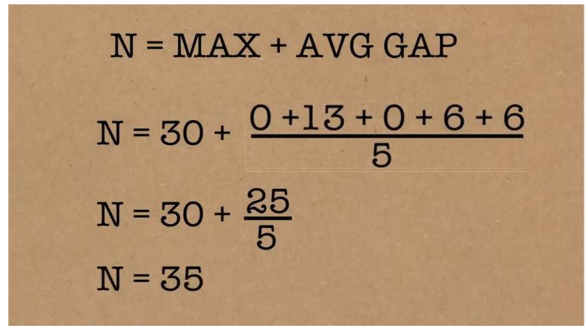


Figure 3

So the issue for me was

How does $(N_{\text{Max}} - k)/k$ in equation (*) represent the average of the gaps?

Consider k samples along the number line (Figure 4).

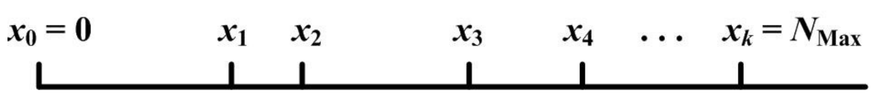


Figure 4

Then the sum of the gaps is

$$(x_1 - x_0 - 1) + (x_2 - x_1 - 1) + (x_3 - x_2 - 1) + \ldots (x_k - x_{k\text{-}1} - 1) = x_k - x_0 - k = N_{\text{Max}} - k,$$

where the gaps don't include the endpoints. Therefore the average is $(N_{\text{Max}} - k)/k$, as desired.

Operations Research

This example provides a window into the origin of "operations research" (OR) that grew out of WWII when mathematicians were called in to analyze military situations. Stephen Budiansky wrote a fascinating book, *Blackett's War* ([2]), about one of the chief practitioners of operations research, Patrick Blackett. From a 2013 book review ([3]):

> Operational research (OR) was essentially the application of common sense and the careful study of data to the messiness of war. The term was first used in a study of how radar could improve Britain's air defenses. As the head of a group of eccentric and eclectic academics known as "Blackett's Circus," Blackett put OR to use to defeat Germany's fearsome U-Boat campaign.

He also considered the problem of how to add a minimum of armor to bombers to reduce their fatality rate on bombing runs. When surviving planes returned, the military looked at where they received the most bullet holes and were considering that those were the regions to reinforce. But Blackett said you should do the opposite. That is, the planes that were lost must have been hit in the areas the surviving planes were not. So those were the areas that should receive more armor. The book is filled with many more easily-understood examples of how mathematics and logic aided the war effort.

References

[1] Grime, James, "The Clever Way to Count Tanks", *Numberphile*, 31 July 2024. (https://www.youtube.com/watch?v=WLCwMRJBhuI)

[2] Budiansky, Stephen, *Blackett's War: The Men Who Defeated the Nazi U-Boats and Brought Science to the Art of Warfare*, Knopf, 2013

[3] Thomas, Evan, " 'Blackett's War: The Men Who Defeated the Nazi U-Boats and Brought Science to the Art of Warfare' by Stephen Budiansky", *Washington Post*, 29 March 2013. (https://www.washingtonpost.com/opinion/blacketts-war-the-men-who-defeated-the-nazi-u-boats-and-brought-science-to-the-art-of-warfare-by-stephen-budiansky/2013/03/29/3083879a-75ee-11e2-8f84-3e4b513b1a13_story.html)