Spy Gift Problem

14 February 2024

Jim Stevenson

This is a recent Alex Bellos problem[1] that supposedly can be solved by 12-year-olds!

Today's problems come from Axiom Maths,[2] a charity that that takes high-attaining primary school children and provides them with maths enrichment during secondary school.

One of Axiom's main activities is to organise 'maths circles', in which small groups of pupils get together to tackle fun problems. Such as the ones below, which are aimed at children aged 11/12, and form the basis for further explorations.

Really Secret Santa

A group of nine secret agents: 001, 002, 003, 004, 005, 006, 007, 008 and 009 have organised a Secret Santa. The instructions are coded, to keep the donors secret.

- *Agent 001 gives a present to the agent who gives a present to agent 002*
- *Agent 002 gives a present to the agent who gives a present to agent 003*
- *Agent 003 gives a present to the agent who gives a present to agent 004*
- *and so on, until*
- *Agent 009 gives a present to the agent who gives a present to agent 001*

Which agent will agent 007 get her present from?

My Solution

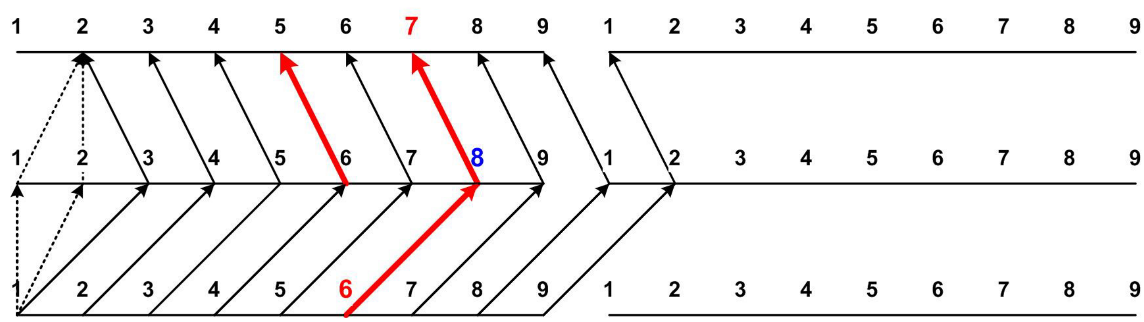


Figure 1

I began by examining possibilities. Figure 1 shows the two step giving sequences described in the problem going from the bottom list of agents to the top list, passing through the intermediate givers. The gift path must end at an agent one number higher than the initial agent, wrapping around from agent 009 to agent 001. Since we assume no agent can give a gift to themselves, the first candidate for agent 001 to give to is agent 003. Then linking the other gift givers incrementing by

[1] https://www.theguardian.com/science/2024/feb/05/can-you-solve-it-are-you-smarter-than-a-12-year-old

[2] https://axiommaths.com/

one shows the path 6 → 8 → 7. But this would also imply 6 → 5 in the upper gift-giving pairing, which is contradictory.

Suppose we shift the intermediate links one to the right, as shown in Figure 2. Then we have the path 6 → 9 → 7, but the contradictory 6 → 4. So we incremented the intermediate agent by one and decremented the contradictory agent by one.

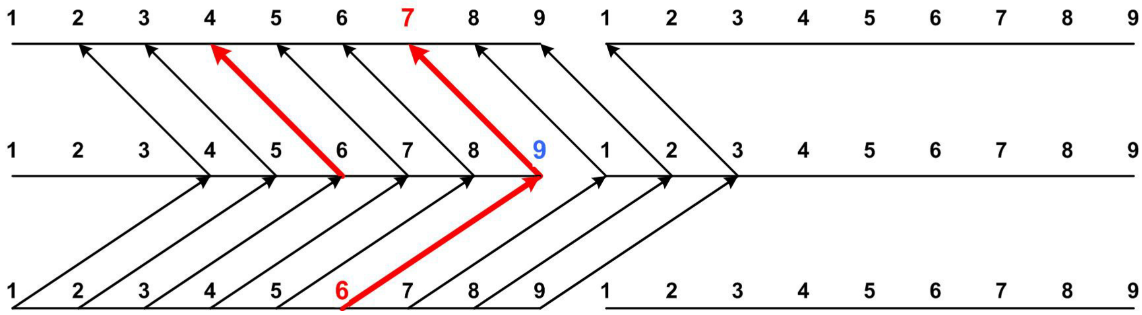


Figure 2

If we do this two more times, we get the result in Figure 3. So now we have 6 → 2 → 7 and 6 → 2, and all the other gift pairings are consistent. So this is the solution.

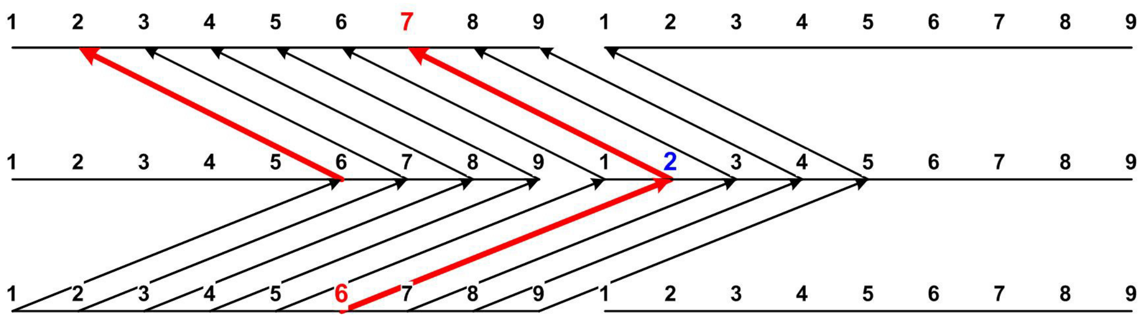


Figure 3

If we express the relationships in Figure 3 differently, we see an interesting pattern (Figure 4).

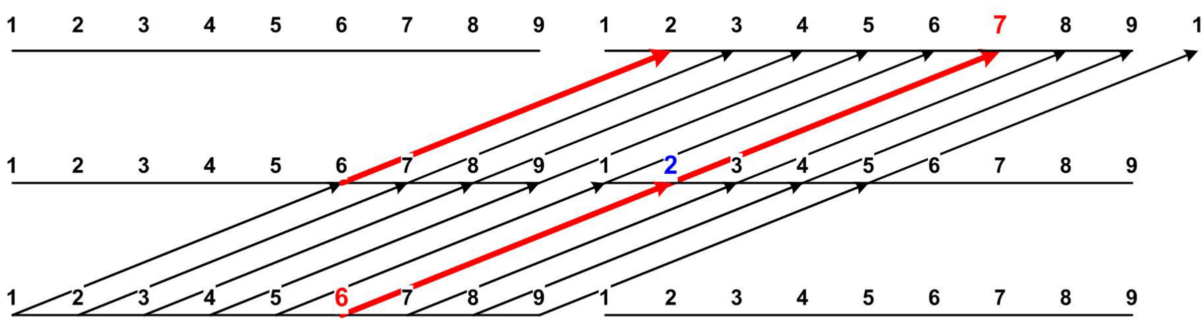
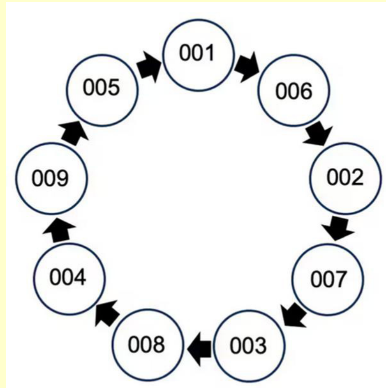


Figure 4

The upper pairings have to be a vertically shifted version of the lower pairings, that is, the pairing arrows have to be parallel to one another and emanate from the same positions as the lower pairings. This means the path from initial giver to final recipient must be a *straight line*. And this means the solution must be *unique*, since each straight line path must end at an agent one unit higher than the starting agent (with the 009 → 001 wrap around).

Bellos Solution[3]

Answer: 002. The easiest way to do this is draw a circle and then fill it in.



What?! Yes, this is the same solution I got, but it was not clear to me at first what Bellos meant by “fill it in.” Then I understood. The idea is to begin filling in every *other* circle with consecutive agent numbers, since that expresses the initial and final relationship between the givers (Figure 5).

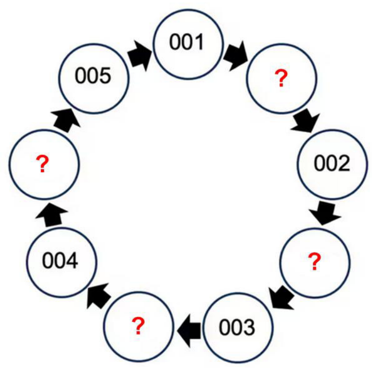


Figure 5

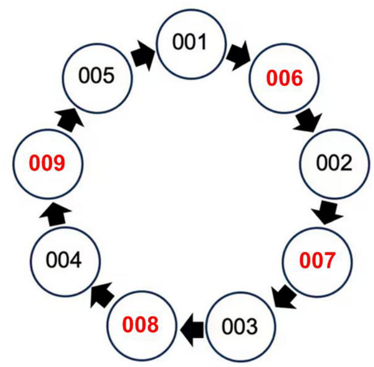


Figure 6

Then “miraculously” the 005 → 006 link just happens to hop over agent 001, and so defines the intermediate gift giver. We continue until all agents are filled in (Figure 6). And, voila! We have the answer. This seems a bit too slick, and it is not quite clear why it works.

What happens if there are only 8 agents? Figure 7 shows the leap-frog method fails. However, if we have 7 agents, then the method works again (Figure 8, Figure 9).

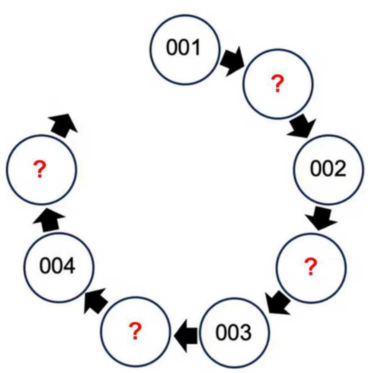


Figure 7 Eight Agents

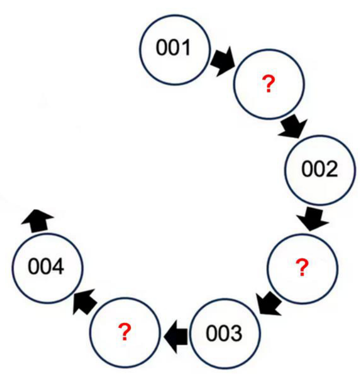


Figure 8 Seven Agents

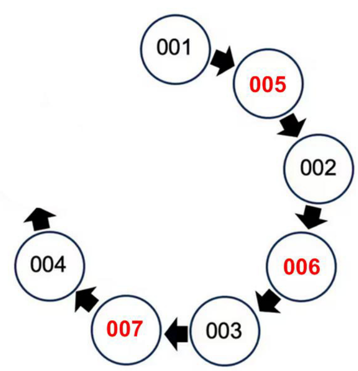


Figure 9 Seven Agents

[3] https://www.theguardian.com/science/2024/feb/05/did-you-solve-it-are-you-smarter-than-a-12-year-old

What is going on? Perhaps viewing the situation from the perspective of my solution will help.

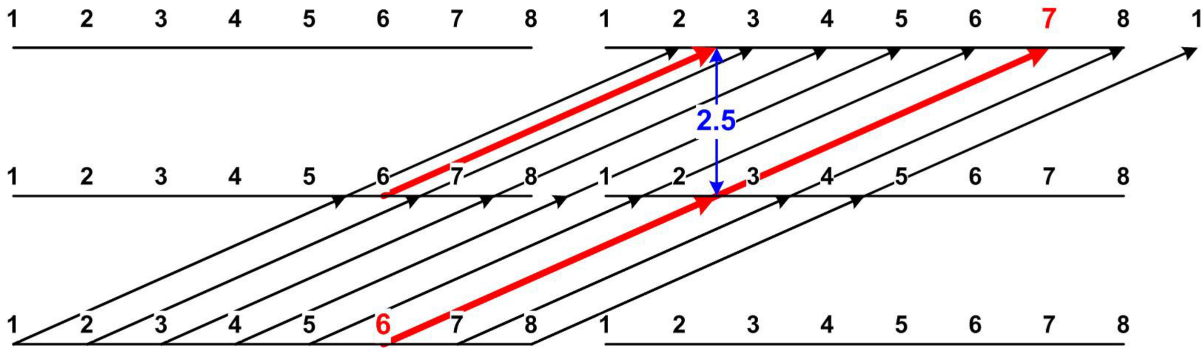


Figure 10 Eight Agents

Since the solution must be represented by straight line paths from initial giver to final recipient, in the case of 8 agents, the intermediate giver for 006 → 007 must be "agent" 002.5 (Figure 10)! But if we have only 7 agents, the intermediate giver for 006 → 007 is the integral agent 003 (Figure 11).

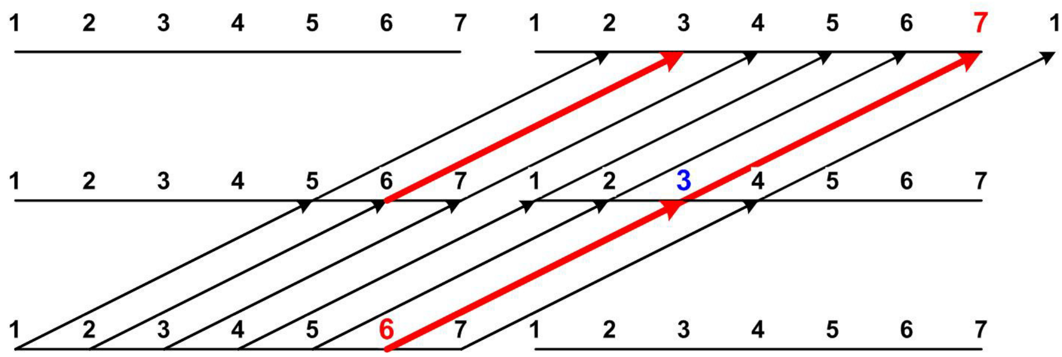


Figure 11 Seven Agents

So the problem can only be solved if there are an odd number of agents, and that allows the circular leap-frog method to work (still somewhat mysteriously to my mind).

That this can be solved by today's 12-year-old fills me with optimism for the future, since I doubt if I could have done it at that age. (I confess I am a bit dubious about this, since my mind was filled with notions of vectors, parallelism, and periodicity—all things that seem to come from "mathematical maturity" based on several years of experience.)