## Yet Another Race

22 October 2022

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This is another race puzzle from the Maths Masters team, Burkard Polster (aka Mathologer) and Marty Ross ([1]) as part of their "Summer Quizzes" offerings for 2013.

In a 100 meter race, Jacob can beat Johann by 5 meters, and Johann can beat Nicolaus by 10 meters. By how much can Jacob beat Nicolaus?

## My Solution

Figure 1 provides a space-time diagram of the problem. Let $\mathrm{v}_{1}, \mathrm{v}_{2}$, and $\mathrm{v}_{3}$ be Jacob's, Johann's, and Nicolalus's speeds respectively. And let $\mathrm{T}_{1}, \mathrm{~T}_{2}$, and $\mathrm{T}_{3}$ be their respective times to finish the 100 m race. Then we have the following equations, based on the "distance $=$ rate $x$ time" model.

$$
\begin{gathered}
\mathrm{v}_{1} \mathrm{~T}_{1}=100 \\
\mathrm{v}_{2} \mathrm{~T}_{1}=95 \\
\mathrm{v}_{2} \mathrm{~T}_{2}=100 \\
\mathrm{v}_{3} \mathrm{~T}_{2}=90 \\
\mathrm{v}_{3} \mathrm{~T}_{3}=100
\end{gathered}
$$



Figure 1

We are trying to find out how far Nicolaus has traveled when Jacob has finished the race, that is, the value of $\mathrm{v}_{3} \mathrm{~T}_{1}$.

The second and third equations yield

$$
\mathrm{T}_{1} / \mathrm{T}_{2}=95 / 100 .
$$

Therefore

$$
\mathrm{v}_{3} \mathrm{~T}_{1}=\mathrm{v}_{3}(95 / 100) \mathrm{T}_{2}=(95 / 100) 90=85.5 \mathrm{~m} .
$$

So Jacob beats Nicolaus by

$$
100-85.5=14.5 \mathrm{~m}
$$

## Maths Masters Solution

Answer: 14.5 meters.
Solution: Suppose Jacob runs at speed $V$. Then Johann runs at speed 19V/20. Since Nicolaus runs at $9 / 10$ of the speed of Jacob, he runs at speed $171 \mathrm{~V} / 200$. So, when Jacob runs 100 meters, Nicolaus will have run 85.5 meters.

## References

[1] Polster, Burkard and Marty Ross, "The Maths Masters' Summer Quiz, Problem Hard 5", The Age, 9 December 2013 https://www.qedcat.com/summerquizzes/2013\ QUIZ.pdf)
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