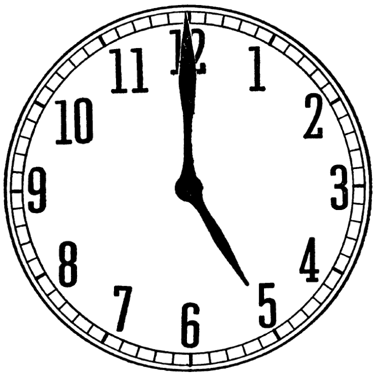
After Five O'clock

19 February 2022

Jim Stevenson



This is a fairly extensive clock problem by Geoffrey Mott-Smith from 1954 ([1]).

The clock shown in the illustration has just struck five. A number of things are going to happen in this next hour, and I am curious to know the exact times.

(a) At what time will the two hands coincide?

(b) At what time will the two hands first stand at right angles to each other?

(c) At one point the hands will stand at an angle of 30 degrees, the minute hand being before the hour hand. Then the former will pass the latter and presently make an angle of 60 degrees on the other side. How much time will elapse between these two events?

My Solution

Again I followed the pattern developed for the "Fallen Clock Puzzle" posting[1] and "A Question of Time" posting.[2] The key is to convert everything into minutes. From those posts:

> Suppose the time is given in h hours and m minutes. First, we wish to find out where the hour hand would be in minutes. Each hour represents 5 minutes on the clock and each minute is 1/60 of an hour and therefore 1/60 of a 5 minute interval. So in terms of minutes, the total time for the hour hand is given by
>
> $$h5 + (m/60)5 = 5h + m/12 \text{ (minutes)}$$
>
> The minute hand is given simply as m minutes.

Problem (a)

So the answer to the first problem for when the hands coincides is given by

$$5 \cdot 5 + m/12 = m$$

or

$$11m = 12 \cdot 25$$

or

$$m = 27.3 \text{ min}$$

or, since converting to fractions gives

$$(12 \cdot 25/11 - 27) = 3(100/11 - 9) = 3/11$$

$$m = 27\ {}^{3}/_{11} \text{ min}$$

Problem (b)

Since a 90° angle corresponds to 15 minutes, we have the following

[1] http://josmfs.net/2020/05/23/fallen-clock-puzzle/

[2] https://josmfs.net/2021/09/18/a-question-of-time/

$$5\cdot 5 + m/12 = m + 15$$

or $$11m = 12\cdot 10$$

or $$m = 12\cdot 10/11 = 10.9 \text{ min}$$

or, since converting to fractions gives

$$(12\cdot 10/11 - 10) = 10(12/11 - 1) = 10/11$$

$$m = 10\ {}^{10}/_{11} \text{ min}$$

Problem (c)

Since a 30° angle corresponds to 5 minutes and a 60° angle to 10 minutes, we want two values for the minutes, m_1 and m_2, corresponding to the two positions of the hands.

$$5\cdot 5 + m_1/12 = m_1 + 5$$

and $$5\cdot 5 + m_2/12 = m_2 - 10$$

Subtracting the first equation from the second gives us a single equation in the desired difference $m_2 - m_1$, namely

$$(m_2 - m_1)/12 = m_2 - m_1 - 15$$

or $$11(m_2 - m_1) = 12\cdot 15$$

or $$m_2 - m_1 = 12\cdot 15/11 = 16.4 \text{ min}$$

or, since converting to fractions gives

$$(12\cdot 15/11 - 16) = 4(45/11 - 4) = 4/11$$

$$m_2 - m_1 = 16\ {}^{4}/_{11} \text{ min}$$

Mott-Smith Solution

These puzzles, like many others concerning clocks, are based on the circumstance that the minute hand travels 12 times as fast as the hour hand. The basic equation for such puzzles is

$$D = 12d$$

where D is the distance traversed by the minute hand and d the distance traversed by the hour hand.

In the ordinary garden variety of clock, the hands move by little jerks. They stand still for an interval of one or two seconds, or even, in many electric clocks, for a full minute. But puzzle books are inhabited by a very special kind of clock in which the hands move continuously.

(a) $27{}^{3}/_{11}$ minutes past five. The second equation here is $D = d + 25\cdot$

(b) $10{}^{10}/_{11}$ minutes past five. At right angles, the hands are 15 minutes apart. Hence D + 15 = d + 25.

(c) $16{}^{4}/_{11}$ minutes. The involved statement of the question is equivalent to asking: What is the interval of time between coincidence of the hands and the next position 90 degrees apart? The answer can be found simply by subtracting the answer to (b) from the answer to (a).

References

[1] Mott-Smith, Geoffrey, "79. After Five O'clock," *Mathematical Puzzles for Beginners & Enthusiasts*, Blakiston Co, 1946, 2nd revised edition, Dover Publications, 1954.