Floating Square Puzzle

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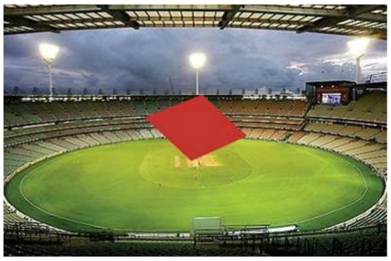

This is another puzzle from the Maths Masters team, Burkard Polster (aka Mathologer) and Marty Ross ([1]) as part of their "Summer Quizzes" offerings.

A mysterious square has materialized in the middle of the MCG, hovering in mid-air. The heights above the ground of three of its corners are 13, 21 and 34 metres. The fourth corner is higher still. How high?

My Solution

I parameterized the problem with vectors (Figure 1). Imagine a three-dimensional coordinate system with the position vector to the lowest corner of the floating square **r** positioned at the origin:

$$\mathbf{r} = 13\,\mathbf{k}.$$

Then vectors along the sides are given by

$$\mathbf{u} = a_1\mathbf{i} + b_1\mathbf{j} + (21 - 13)\mathbf{k}$$

$$\mathbf{v} = a_2\mathbf{i} + b_2\mathbf{j} + (34 - 13)\mathbf{k}$$

Since the shape is a square, each side is parallel to, and the same length as, its opposite side. Therefore the vectors are parallel translates of each other as shown.

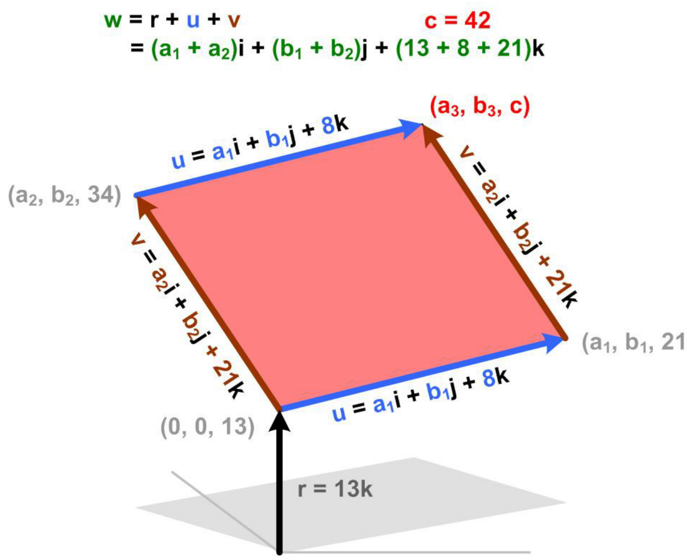

Figure 1

The height of the fourth corner of the square is obtained as the third component of the vector **w**:

$$\mathbf{w} = \mathbf{r} + \mathbf{u} + \mathbf{v} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j} + (13 + 8 + 21)\mathbf{k} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j} + 42\mathbf{k}$$

Recall that vectors add component-wise, so, since we are only interested in the vertical component, it doesn't matter what the **i**, **j** components are. Thus the answer is 42 meters.

Maths Masters Solution

The Maths Masters' solution is equivalent (Figure 2).

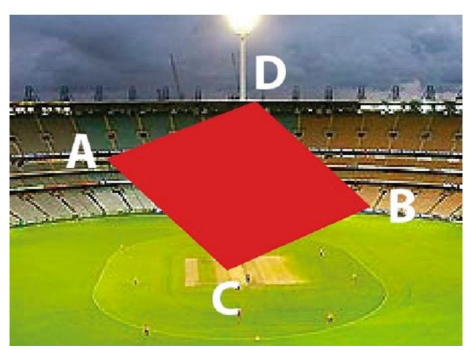

Figure 2

Answer. The height of the fourth corner is 42 meters, as readers of *The Hitchhikers Guide to the Galaxy* would have already guessed.

Solution. Label the corners *A*, *B*, *C* and *D*, as in the diagram. Note that the rise from *B* to *D* equals the rise from *C* to *A*. (To see this, all that matters is that our shape is a parallelogram). So, using the letters to stand for the heights of the corners,

$$D - C = (D - B) + (B - C) = (A - C) + (B - C).$$

Using the values of A, B, and C, we find $D = 42$ meters.

References

[1] Polster, Burkard and Marty Ross, “Maths Challenge 2009, Problem 21”, *The Age*, 30 November 2009 (https://www.qedcat.com/summerquizzes/2009%20QUIZ.pdf)