The Lure of Mathematics Conundrum

31 July 2019, updated 28 July 2023

Jim Stevenson

agsandrew / Fotolia

A prevalent theme of much of popular mathematical exposition and debates about mathematics education concerns how to interest a wider population in matters mathematical. For the most part I feel that essays that try to present the "beauty" of mathematics are doomed to failure, as are most discussions of esthetics. The underlying goal of such writing is a legitimate and laudable attempt to show the appeal of math. But I fear it succeeds only with those already converted.

Appeals to the utility of math are also dubious since, other than arithmetic, percentages, and some elementary probability and statistics, most people will probably not be required to apply very sophisticated mathematics in their lives.

Contributing to this common lack of interest in mathematics, and in fact often causing active resistance or repugnance, is the complaint that students do not "understand" mathematics and it is all just a set of rules one has to "memorize." So the great understanding vs. memorization debate was born. Our difficulty will be solved, then, and students will come to appreciate mathematics in their early years if they can just be brought to "understand" it. That too is doomed to failure.

I should make clear again that I am not active in math education or research and view the latest studies from the outside like most everyone else. But a lot of the conversation about math education centers on the future, that is, how to prepare the eventual general citizen or professional technologist. To that I can give my personal response to what happens to a person in the future who became involved in mathematics.

A while ago I wrote an essay on why I became a mathematician that I might have the nerve to publish someday, it being of a more personal nature. When I reflect on that essay, I see it is more a narrative of *how* I became a mathematician rather than *why*. But there are some nuggets in that story that I have thought about more deeply that may be relevant to the general question of how to interest others in math. In particular, I disliked math, that is, arithmetic, very much in elementary school, as well as the jumbled assortment of topics presented in junior high in our day up to the advent of algebra (in ninth grade). Algebra proved more interesting and plane geometry was what really brought home the excitement of math to me. I know things have changed somewhat, but there may be some invariants here that need to be considered.

I sense that modern efforts to make elementary school children "understand" the math and therefore find it less repellent will likely fail. To me "understanding" means to relate the new thing to something you already know, as well as to explore all its behavior and applications in new settings, usually through many examples. But young children don't have this wealth of experience yet. For children, especially, learning is inductive from a build-up of experiences. This is guided by their parents and other adults to whom they should defer as more knowledgeable. This utopian scenario is captured in the time honored dialog of "Why, why? … Because I told you so" as every frustrated parent has experienced. But I believe this is a fundamental invariant of growth. Children just have to be taught some basics from their parents and trust that they know it will be necessary. They need to learn the fundamentals of arithmetic and apply them in many situations to build up that collection of experiences—and that is a largely "procedural" task (I think the latest rubric is "mathematical fluency" [1]). It is only as teenagers (middle school and high school) that a child's natural rebellion and challenge mean they are ready to really understand for themselves. Almost every mathematician I knew came to the field in high school.

When I grew up, I just accepted that the basic Rs (Reading, Writing, 'Rithmetic) were essentials of education for everybody and you just did it. I may not have understood why, but I did appreciate that they were considered essential. In fact, the language side of education suffered from similar demands to learn basics, such as spelling, vocabulary, sentence structure. Another parallel to this is music. We have to learn fingering for a particular instrument, as well as how to read music. We then play simple scales until our fingers begin to move without thought. But as soon as possible we try to play tunes. The enjoyment from that experience provides motivation to improve again through dull practice so we can play even better tunes. Similarly with the language skills: soon we are reading stories ourselves (after having them read to us) or gaining information through books independently of our parents. But with math, this type of reward is usually much more limited or non-existent. So something needs to take its place until an appreciation for real math can develop.

By high school most students are already turned off by math. So what to do? Why isn't everyone discouraged? After having one of these early repellent experiences with arithmetic, why did I ultimately find math appealing? I believe the answer does not lie with mathematics per se—its explicit manipulation of numbers—but with a broader urge to understand, and something more—magic! I mean magic tricks. A child has to reach a certain age before they appreciate a magic trick. When they are really young, the whole world is a magic trick. Unexplained and startling things are happening to them all the time and they are avidly trying to organize their experiences into some consistent pattern. That is the joy of watching a grandchild master this situation. At some point, maybe 6 or 7 years old, the child has enough of a stable view of "reality" to appreciate a deviation from the expected. They finally appreciate a magic trick. I became infatuated and seduced by magic in fourth grade and never stopped. I have never met a child who doesn't go bananas when you make a coin disappear (French drop) and then reappear from their ear. Their grip on reality is firm enough that they *know* that is impossible—there must be a *rational* explanation. And there it is. That is the key to the lure of mathematics and every other endeavor involving finding rational explanations for things. Not everyone is willing to devote their lives to such pursuits, but a vast number are willing to stop periodically and share the results of those who have. Television is filled with shows investigating the "mysteries" of the past or even the present with questions about objects that have been revealed by satellites or Google maps. There is even a proliferation of science shows that attempt to bring the mysteries of the cosmos or human biology into the public view.

Of course, we all can't be magicians, but education can focus us on the "questions" that are the magic tricks of nature or history. This can take various forms. First you have to interest a child in the elements of nature or history. (That may not be so hard since their natural curiosity is already asking interminable "whys".) This can come from questions of what were things like in the past—the distant past of dinosaurs or prehistoric peoples or the recent past of ancient civilizations and the making of nations, including our own.

What does this all have to do with interest in math? It is a suggestion to stimulate the mathematics urge by means other than explicit mathematics, that is, through the fostering of curiosity and the desire to find logical answers to intriguing questions: where do things come from? how do they make something? what determines the branching in plants and trees? how does the embryo develop? what is money (is gold better then paper money)? How would you go about answering these questions (and maybe more suitable questions for a younger age)? How would you know if something is true or not? And if you know things, can you predict the future with that knowledge? Having pursued these questions in non-mathematical areas a student is primed to find mathematics as a tool or model to help answer the questions. Mathematics has its own criterion for truth and it has a powerful predictive factor—if it is applied carefully and knowledgeably. Then a student can decide whether they want to develop these tools better (become a mathematician) or apply them more widely and effectively (become a physicist, chemist, biologist, economist, engineer, or more adept user of technology).

And on a fundamental level of being an educated person, there is Feynman's update of Galileo's admonishment:

> I really think that … two cultures separate people who have and people who have not had this experience of understanding mathematics well enough to appreciate nature once. It is too bad that it has to be mathematics, and that mathematics is hard for some people. … Physicists cannot make a conversion to any other language. If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in. She offers her information only in one form; we are not so unhumble as to demand that she change before we pay any attention. ([2] p.58)

Perhaps this is too general, sweeping, and vague. I claim the mathematical urge manifests itself in the joy of solving puzzles and playing games. Everyone likes to do this. Of course, again in the age before television and other electronic devices, puzzles came in the form of "fair" mystery stories, crossword and acrostic puzzles, and jig-saw puzzles (put together without looking at the picture). Many games were largely based on playing cards, which are a marvelous and stupendous invention. The permutations are astounding and the skills required to play many card games require logic, strategy, memory, and imagination. Card games are really quite abstract and not far at all from the pleasure found in mathematics.

So I guess my mathematical education fantasy for children entails early mastery of the rudiments of the three Rs, guided by the faith that elders have determined it is important for survival. Good teachers can make the arithmetic chores palatable in a number of ways, using objects to support learning by doing. In parallel, good basic education should foster curiosity through appealing questions and teach the analytic methods for exploring and answering these questions through concrete projects. These methods should also include justifying your answers, that is, how do you know something is so. I believe these activities would provide the basis for the general public to appreciate mathematics and a motivated subset to pursue either math itself or math-supported subjects (which is virtually every subject nowadays).

Postscript (7/24/2023)

This essay has sat in the can for four years—mainly because who am I to pontificate on current math education, since I am so long removed from the endeavor. However, that does not mean I have no experience in the matter, especially since I have been blessed with having an advanced education in math and using it throughout my professional and personal life.

From what I could glean from the Twitter accounts of modern public school educators, there has been vast improvement in educating students in the basics of math, especially compared to my day. But the numerous essays by David Bressoud[1] and others indicate there is a strong debate concerning preparation for more advanced mathematics in the public schools that will support both the general public and those going on to more specialized technical endeavors, especially those requiring calculus. In Bressoud's latest essay, "Popular Perceptions of Mathematics Instruction" ([3]), a study of how parents and teachers feel about current math education indicated that two of the most important views were: "Making math education more real-world applicable" and "Teaching more practical life skills in math class." Bressoud summarizes the situation by saying:

[1] https://mathvalues.squarespace.com/launchings

> What does this mean? In short there is a lot of sympathy for updating the curriculum to include statistics, data analysis, and quantitative reasoning. However, there are still many obstacles to making this happen. We need to prepare more teachers who are well-equipped to teach mathematics with these emphases. We need to convince parents that providing resources to build programs in these areas will not limit access to traditional accelerated preparation for calculus. And most importantly, we must guarantee that programs in statistics, data analysis, quantitative reasoning, or other non-traditional paths do not become dead ends, that the student who starts on one of these programs is not disadvantaged if they later decide to pursue a college major that requires calculus.

A constant drumbeat in these discussions is the emphasis on "statistics, data analysis, and quantitative reasoning" as the most important way to make math "useful". Historically mathematics arose out of the necessity to solve practical problems in business, commerce, manufacturing, navigation, and vast areas covered by physics. The difficulty is that solving problems in the real world is very hard. As I have mentioned before, real world problems are extreme "word problems", which have always been a challenge. In particular, problems in probability and statistics are notoriously difficult to understand. Since probabilities involve percentages, I have often thought they were similar to the difficulties in understanding mixture problems in algebra, which were the hardest type of word problems. So emphasizing "statistics, data analysis, and quantitative reasoning" is a non-trivial task. In many ways it harkens back to the horrible problems I had in trig classes measuring the distances across swamps with trig tables and logarithms. Mind-numbing and boring.

The modern era, however, has some tools that are fantastic and remove much of the onus of mindless calculations, namely, computational aids on personal computers such as spreadsheets like Excel and analysis tools like Igor and Matlab. These tools are invaluable for anyone to learn, whether they are going into a technical field or not. Trying to work out the comparative benefits of different prescription or medical plans would be impossible without the aid of a spreadsheet. Statistics takes the measure of a static situation that has already occurred. Hopefully, the results may be extrapolated into the future, but such predictive capability really relies on some type of dynamic model, and that is where calculus comes in, especially in the form of differential equations. Still, there are finite-difference approximations that lend themselves to computer simulations that can easily be handled in Matlab and Igor, or even more sophisticated programming languages. The advantage of Excel, Igor, and Matlab is that they include integrated plotting components that let you visualize the results. An interesting example of the scope of this hands-on approach is given in the 1994 book *Calculus in Context*, which is full of simulation examples from biology, physics, and economics.

So I just wanted to caution that limiting mathematics instruction to what is "useful" may be harder than one thinks, but with modern tools, there may be still be some success, even for the less motivated student.

References

[1] Gowers, Tim, "How Craig Barton Wishes He'd Taught Maths", *Gower's Weblog*, 22 December 2018 (https://gowers.wordpress.com/2018/12/22/how-craig-barton-wishes-hed-taught-maths/#more-6435, retrieved 1/13/2019)

[2] Feynman, Richard, *The Character of Physical Law,* MIT Press, 1965. First MIT Press Paperback Edition, March 1967, 12th printing, 1985.

[3] Bressoud, David, "Popular Perceptions of Mathematics Instruction", *Launchings, Teaching & Learning*, 1 July 2023. (https://mathvalues.squarespace.com/masterblog/popular-perceptions-of-mathematics-instruction, retrieved 7/13/2023)