Al the Chemist II

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This is the second part of the problem from Raymond Smullyan in the "Brain Bogglers" section of the 1996 *Discover* magazine ([1]).

> On another occasion, Al made a mixture of water and wine. There was more water than wine—in fact, the excess of water over wine was equal to one-fourth the quantity of wine. Al then added 12 ounces of wine, at which point there was one ounce more of wine than water.
>
> According to another version of the story, before Al added the 12 ounces of wine, he first boiled off 12 ounces of water (the net effect being that he replaced 12 ounces of water with wine), and again there was one more ounce of wine than water.
>
> Would there be more mixture present at the end of the first version or the second?

I found this statement a tad ambiguous with the result that I found two possible solutions: the one Smullyan gave and another, surprising one.

Solution

Let w be the original volume of water in the container and v the original volume of wine (vino) in the container. The common part to both versions of the story claimed that

$$w - v = ¼\, v \ \text{ or } \ w = {}^5/_4\, v$$

The first version of the story then said that

$$(v + 12) - w = 1$$

This means

$$w - v = 11 = ¼\, v,$$

so

$$v = 44 \text{ oz } \text{ and } w = {}^5/_4\, 44 = 55 \text{ oz.}$$

Therefore the amount of mixture at the end is

$$(v + 12) + w = 56 + 55 = 111 \text{ oz.}$$

Smullyan Interpretation.

For the second version of the story we can assume, as Smullyan did, that there are 12 oz. less of the water when we add the 12 oz. of wine, yielding

$$(v + 12) - (w - 12) = 1$$

or

$$w - v = 23 = ¼\, v$$

so

$$v = 92 \text{ oz. } \text{ and } w = {}^5/_4\, 92 = 115 \text{ oz.}$$

Therefore the amount of mixture at the end is

$$(v + 12) + (w - 12) = w + v = 92 + 115 = 207 \text{ oz.}$$

which is greater than the 111 oz. in the first version.

My Alternative Interpretation.

Trying to read the text very carefully, I assumed when the water was boiled off the mixture, that meant the whole mixture was heated and some wine was boiled off as well. Let x be the amount of mixture boiled off to obtain a loss of 12 oz. of water (assuming the "water" in the wine is counted as wine). I further assumed that boiling the mixture would maintain the proportions of water and wine in the mixture.

Let V be the volume of the original mixture, namely $V = w + v$. Then the fraction of water in the mixture is

$$w / (w + v) = {}^5/_4\, v / ({}^5/_4\, v + v) = {}^5/_9$$

and so the fraction of wine in the mixture is ${}^4/_9$. Therefore, boiling off x amount of liquid yields

$$V - x = {}^5/_9\, (V - x) + {}^4/_9\, (V - x) = (w - {}^5/_9\, x) + (v - {}^4/_9\, x)$$

From the second version of the story we have the amount of water boiled off is 12 oz., so

$${}^5/_9\, x = 12 \text{ or } x = {}^9/_5\, 12 \text{ oz.}$$

Therefore, the amount of wine boiled off is

$${}^4/_9\, x = {}^4/_9\, {}^9/_5\, 12 = {}^4/_5\, 12 \text{ oz.}$$

So my interpretation of version two of the story would say

$$((v - {}^4/_5\, 12) + 12) - (w - 12) = 1$$

or

$$w - v = {}^6/_5\, 12 - 1 = {}^{67}/_5 = ¼\, v$$

so

$$v = {}^4/_5\, 67 = 53.6 \text{ oz. and } w = = {}^5/_4\, {}^4/_5\, 67 = 67 \text{ oz.}$$

Therefore the amount of mixture at the end is

$$(v + {}^1/_5\, 12) + (w - 12) = (53.6 + 2.4) + 55 = 56 + 55 = 111 \text{ oz.}$$

which is the *same* as in the first version of the story! Totally unexpected.

References

[1] Smullyan, Raymond, "Brain Bogglers: Al the Chemist", *Discover*, August 1996