Valid Argument

Copi, Irving M., Carl Cohen, and Kenneth McMahon, *Introduction to Logic*, 14th ed., Pearson Education Limited, 2014 (angg.twu.net/tmp/2016-optativa/copi__introduction_to_logic.pdf)

JOS: Text below that appears in a yellow box was presented in the margins of the original book. Words in bold or italics are as they appeared in the original text. Page numbers in parentheses correspond to the location in the PDF version.

[p.6 (11)]:

Argument

Any group of propositions of which one is claimed to follow from the others, which are regarded as providing support or grounds for the truth of that one.

Conclusion

In any argument, the proposition to which the other propositions in the argument are claimed to give support, or for which they are given as reasons.

Premises

In an argument, the propositions upon which inference is based; the propositions that are claimed to provide grounds or reasons for the conclusion.

Argument is a technical term in logic. It need not involve disagreement, or controversy. In logic, **argument** refers strictly to any group of propositions of which one is claimed to follow from the others, which are regarded as providing support for the truth of that one. For every possible inference there is a corresponding argument.

In writing or in speech, a passage will often contain several related propositions and yet contain no argument. An argument is not merely a collection of propositions; it is a cluster with a structure that captures or exhibits some inference. We describe this structure with the terms *conclusion* and *premise*. The **conclusion** of an argument is the proposition that is affirmed on the basis of the other propositions of the argument. Those other propositions, which are affirmed (or assumed) as providing support for the conclusion, are the **premises** of the argument.

[p.329 (354)]:

Variable or statement variable

A place-holder; a letter (by convention, any of the lower case letters, beginning with p, q, etc.) for which a statement may be substituted.

Argument form

An array of symbols exhibiting logical structure; it contains no statements but it contains statement variables. These variables are arranged in such a way that when statements are consistently substituted for the statement variables, the result is an argument.

Substitution instance

Any argument that results from the substitution of statements for the statement variables of a given argument form.

We define an **argument form** as any array of symbols containing statement variables but no statements, such that when statements are substituted for the statement variables—the same statement being substituted for the same statement variable throughout—the result is an argument. For definiteness, we establish the convention that in any argument form, *p* shall be the first statement variable that occurs in it, and as other variables are introduced, they shall be labeled *q*, *r*, and *s*. Thus the expression

$$p \Rightarrow q$$

$$\underline{q\quad\quad}$$

$$\therefore p$$

is an argument form,[1] for when the statements *B* and *G* are substituted for the statement variables *p* and *q*, respectively, the result is the first argument in this section. If the statements *A* and *D* are substituted for the variables *p* and *q*, the result is the second argument. Any argument that results from the substitution of statements for statement variables in an argument form is called a **substitution instance** of that argument form. Any substitution instance of an argument form may be said to have that form, and any argument that has a certain form is said to be a substitution instance of that form.

[p.332 (357)]:

One can proceed by relying upon the technique of refutation by logical analogy. The term **invalid** as applied to argument forms may be defined as follows: *An argument form is invalid if and only if it has at least one substitution instance with true premises and a false conclusion*. If the specific form of a given argument has any substitution instance whose premises are true and whose conclusion is false, then the given argument is invalid. This fact—that any argument whose specific form is an invalid argument form is an invalid argument—provides the basis for refutation by logical analogy. A given argument is proved invalid if a refuting analogy can be found for it.

"Thinking up" refuting analogies may not always be easy. Happily, it is not necessary, because for arguments of this type there is a simpler, purely mechanical test based on the same principle. Given any argument, we can test the specific form of that argument, because its invalidity would determine the invalidity of the argument.

The test described above can also be used to show validity. Any argument form that is not invalid must be valid. Hence *an argument form is* **valid** *if and only if it has no substitution instances with true premises and a false conclusion*. Because validity is a formal notion, an argument is valid if and only if the specific form of that argument is a *valid argument form*.

Testing Argument Validity Using Truth Tables

Knowing exactly what it means to say that an argument is valid, or invalid, we can now devise a method for testing the validity of every truth-functional argument. Our method, using a truth table, is very simple and very powerful. It is simply an application of the analysis of argument forms just given. To test an argument form, we examine all possible substitution instances of it to see if any one of them has true premises and a false conclusion. Of course, any argument form has an infinite number of substitution instances, but we need not worry about having to examine them one at a time. We are interested [p.333 (358)] only in the truth or falsehood of their premises and conclusions, so we need consider only the truth values involved. The arguments that concern us here contain only simple statements and compound statements that are built up out of simple statements using the curl [tilde] and the truth-functional connectives symbolized by the dot, wedge, and horseshoe. Hence we obtain all possible substitution instances whose premises and conclusions have different truth values by examining all possible different arrangements of truth values for the statements that can be substituted for the different statement variables in the argument form to be tested.

[1] JOS: In general we can write an argument form as $p_1, p_2, p_3, \ldots, p_n \vdash q$, where the p_i's are the premises and q is the conclusion.

[p.334 (358)]:

…*Is there any one case*, we ask ourselves, *any single row in which all the premises are true and the conclusion false?* If there is such a row, the argument form is invalid; if there is no such row, the argument form must be valid. After the full array has been neatly and accurately set forth, great care in reading the truth table accurately is of the utmost importance.[2]

[p.343 (368)]:

Tautology A statement form all of whose substitution instances must be true.

[p.345 (370)]:

D. Arguments, Conditional Statements, and Tautologies

To every argument there corresponds a conditional statement whose antecedent is the conjunction of the argument's premises and whose consequent is the argument's conclusion. Thus, an argument having the form of *modus ponens*, [p.346 (371)]

$p \Rightarrow q$

$\underline{p\quad\quad}$

$\therefore q$

may be expressed as a conditional statement of the form $((p \Rightarrow q) \wedge p) \Rightarrow q$.[3] If the argument expressed as a conditional has a valid argument form, then its conclusion must in every case follow from its premises, and therefore the conditional statement of it may be shown on a truth table to be a tautology. That is, the statement that the conjunction of the premises implies the conclusion will (if the argument is valid) have all and only true instances.

Truth tables are powerful devices for the evaluation of arguments. An argument form is valid if and only if its truth table has a **T** under the conclusion in every row in which there are **T**'s under all of its premises. This follows from the precise meaning of *validity*. Now, if the conditional statement expressing that argument form is made the heading of one column of the truth table, an **F** can occur in that column only in a row in which there are **T**'s under all the premises and an **F** under the conclusion. But there will be no such row if the argument is valid. Hence only **T**'s will occur under a conditional statement that corresponds to a valid argument, and that conditional statement *must* be a tautology. We may therefore say that an argument form is valid if, and only if, its expression in the form of a conditional statement (of which the antecedent is the conjunction of the premises of the given argument form, and the consequent is the conclusion of the given argument form) is a tautology.

For every *invalid* argument of the truth-functional variety, however, the corresponding conditional statement will not be a tautology. The statement that the conjunction of its premises implies its conclusion is (for an invalid argument) either contingent or contradictory.

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[2] JOS: This truth table approach is equivalent to the statement in Copi, *Symbolic Logic* (1954), p.30: "To every argument corresponds a conditional statement whose antecedent is the conjunction of the argument's premises and whose consequent is the argument's conclusion. That corresponding conditional is a tautology if and only if the argument is valid." That is, $p_1, p_2, p_3, \ldots, p_n \vdash q$ is a valid argument if and only if $P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n \Rightarrow Q$ is a tautology (ascertained via a truth table) for the statements substituted for the variables. The presentation here in this book is a bit obscure. Actually, this book does address this idea, only later, beginning on p.345, as shown below.

[3] JOS: See my truth table for this example in the box below.

JOS: Example of Modus Ponens

Consider the argument Modus Ponens: $\mathbf{p \Rightarrow q,\ p \vdash q}$. The corresponding proposition is $\mathbf{((P \Rightarrow Q) \wedge Q) \Rightarrow Q}$. First we fill in the corresponding truth table with all the four combinations of T and F for P and Q:

(P	$\Rightarrow$	Q)	$\wedge$	P)	$\Rightarrow$	Q
T		T		T		T
T		F		T		F
F		T		F		T
F		F		F		F

Then we fill in the derived truth values (green) for the implication ($\Rightarrow$) in the premises, followed by the derived truth values (magenta) for the conjunction ($\wedge$), and finally the derived truth values (red) for the ultimate implication ($\Rightarrow$):

((P	$\Rightarrow$	Q)	$\wedge$	P)	$\Rightarrow$	Q
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	F	F	T	T
F	T	F	F	F	T	F

And so we see that we obtain a tautology. So the argument is valid.

The sentence above "*an argument form is* **valid** *if and only if it has no substitution instances with true premises and a false conclusion*" means we have eliminated the only case where the ultimate implication ($\Rightarrow$) is false. So we are only left with T values, and hence a tautology.