Bailing Water Problem

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This is a straight-forward problem from *Five Hundred Mathematical Challenges* ([1]).

A boat has sprung a leak. Water is coming in at a uniform rate and some has already accumulated when the leak is detected. At this point, 12 men of equal skill can pump the boat dry in 3 hours, while 5 men require 10 hours. How many men are needed to pump it dry in 2 hours?

My Solution

W_0 = amount of water already in boat
v = rate water is coming in, gal/hr
t = time to pump out water, hr
m = number of men pumping
r = gallons of water pumped per man per hour

Then the mathematical model[1] is

$$W_0 + vt = rmt$$

We are given 12 men can pump out the water in 3 hours, so

$$W_0 + v\cdot 3 = r\cdot 12\cdot 3 \tag{1}$$

and 5 men can pump out the water in 10 hours, so

$$W_0 + v\cdot 10 = r\cdot 5\cdot 10 \tag{2}$$

We are asked how many men m can pump out the water in 2 hours, so

$$W_0 + v\cdot 2 = rm\cdot 2 \tag{3}$$

Subtracting equation (2) from equation (1) yields

$$7v = 14r \;\Rightarrow\; v/r = 2$$

And subtracting equation (3) from equation (1) yields

$$v = r(36 - 2m) \;\Rightarrow\; 2m = 36 - 2 \;\Rightarrow\; m = 17.$$

So it will take 17 men.

Math Challenge Solution

The Math Challenge solution is essentially the same.

Let x be the amount of water present when the pumping begins, y the amount leaking in per hour and z the amount each man can remove per hour. Suppose $h(n)$ is the amount of time in hours needed by n men to pump the boat dry. Then

[1] For a more detailed discussion about models of this type see the 6/9/2019 posting "Fibonacci, Chickens, and Proportions" (http://josmfs.net/2019/09/06/fibonacci-chickens-and-proportions/).

$$x + h(n)y = n\,h(n). \qquad (*)$$

In particular,

$$x + 3y = 12\cdot 3\cdot z = 36\,z,$$

and

$$x + 10y = 5\cdot 10\cdot z = 50\,z,$$

whence $y = 2z$ and $x = 30z$. Thus (*) becomes

$$30 + 2h(n) = n\,h(n) \quad \text{or} \quad h(n)(n-2) = 30.$$

When $h(n) = 2$, $n = 17$ and 17 men are needed to do the job in 2 hours.

References

[1] Barbeau, Edward J., Murray S. Klamkin, William O. J. Moser, "Problem 216", *Five Hundred Mathematical Challenges*, Spectrum Series, Mathematical Association of America, Washington D.C, 1995, p.19.