Rolling Wheels Puzzle

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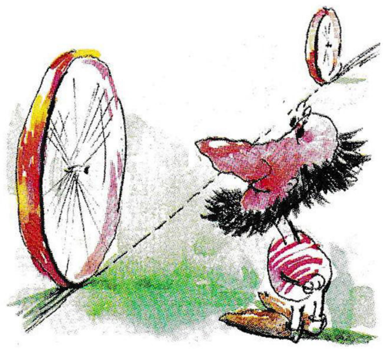
Jim Stevenson



Pavel Chernusky

Here is another *Quantum* math magazine Brainteaser ([1]).

Two wheels roll toward each other with identical angular velocity. At the moment of collision they contact each other at the same points that touched the ground before they began rolling. Could the radii of the wheels differ?

My Solution

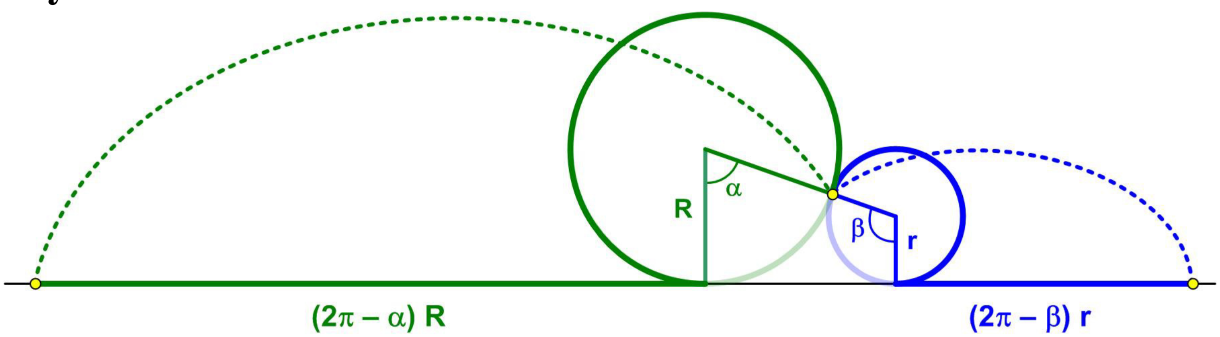


Figure 1 My Solution

Figure 1 shows the problem situation. When the two circles touch, their tangents coincide and so their radii are parallel, and in fact because of their common tangent point they form a straight line between the two centers. Therefore the angles α and β satisfy

$$\alpha + \beta = \pi$$

In addition, the problem states that the points of tangency are the original points of rest on the horizontal line before they rolled together. Therefore, the circles (centers) have each traveled distances $(2\pi - \alpha)R$ and $(2\pi - \beta)r$, respectively. Furthermore, the problem states that their angular rotations must be the same. Thus,

$$2\pi - \alpha = 2\pi - \beta \;\Rightarrow\; \alpha = \beta$$

But that means $$\alpha = \beta = \pi/2$$

and that can happen only if $$R = r.$$

So the radii of the two wheels cannot differ.

Quantum Solution

Look at the angles between the vertical radius of each circle when they intersect and the radius that was originally vertical. It is given that the arcs of these two central angles are equal, so the angles themselves must be equal. But this is impossible if the radii are not equal (Figure 2).

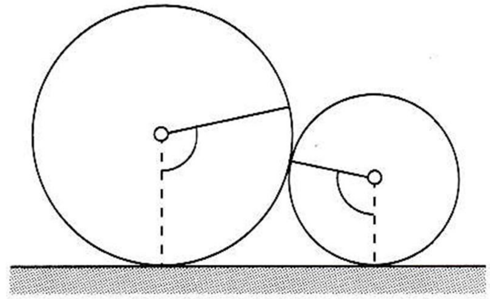


Figure 2 ***Quantum* Solution**

References

[1] “Touching Points” B225 “Brainteasers” Quantum Vol.8, No.3, National Science Teachers Assoc., Springer-Verlag, Jan-Feb 1998. p.9