Who Did It?

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This is a fun logic problem from the *Mathigon* math calendar for December 2022.[1]

1. Alice said Bob did it.

2. Bob said Alice did it.

3. Carol said Alice didn't do it.

4. Dan said it was either Alice or Carol.

Only one person is telling the truth. Who did it?

Solution

As with the other logic puzzles,[2] I like to convert the statements to symbolic logic. So consider the following assignments.

A_1 = Alice is telling the truth. A_2 = Alice did it.
B_1 = Bob is telling the truth. B_2 = Bob did it.
C_1 = Carol is telling the truth. C_2 = Carol did it.
D_1 = Dan is telling the truth. D_2 = Dan did it (assuming *somebody* did it).

Then the four statements in the problem translate into symbolic logic as shown in the table.

Stmt	Implication	Contrapositive
1.	$A_1 \Rightarrow B_2$	$\sim B_2 \Rightarrow \sim A_1$
2.	$B_1 \Rightarrow A_2$	$\sim A_2 \Rightarrow \sim B_1$
3.	$C_1 \Rightarrow \sim A_2$	$A_2 \Rightarrow \sim C_1$
4.	$D_1 \Rightarrow (A_2 \vee C_2)$	$(\sim A_2 \wedge \sim C_2) \Rightarrow \sim D_1$

Recall that the contrapositive is equivalent to the original implication, that is, for statements P and Q, "if P is true, then Q is true" ($P \Rightarrow Q$) is logically equivalent to "if Q is false, then P is false" ($\sim Q \Rightarrow \sim P$). (Again the contrapositive $\sim Q \Rightarrow \sim P$ is not the same as the converse $Q \Rightarrow P$.)

We see from the table via the contrapositive that if A_2, B_2, and C_2 are false (Alice, Bob, and Carol didn't do it), then A_1, B_1, and D_1 are false (Alice, Bob, and Dan are not telling the truth) and C_1 is not false, that is, C_1 is true (Carol is telling the truth)—the only one of the four telling the truth. Therefore D_2 is true, that is, Dan did it (assuming somebody among the four individuals did it).

I haven't been able to see *Mathigon*'s solution.

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[1] https://mathigon.org/puzzles#2022

[2] See for example, Pointing Fingers (http://josmfs.net/2020/09/19/pointing-fingers/), Swallowing Elephants (http://josmfs.net/2020/10/03/swallowing-elephants/), and Do I Avoid Kangaroos (https://josmfs.net/2020/12/05/do-i-avoid-kangaroos/)