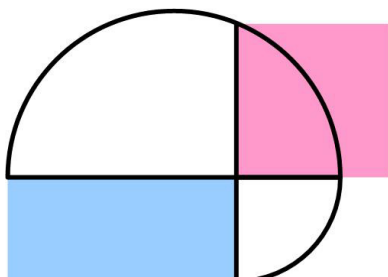


# Spiral Areas Puzzle

31 January 2022

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This is a provocative puzzle from the Maths Masters team, Burkard Polster (aka Mathologer) and Marty Ross ([1]) as part of their “Summer Quizzes” offerings for 2013.

In the picture the top curve is a semicircle and the bottom curve is a quarter circle. Which has greater area, the red square or the blue rectangle?

## My Solution

This was another problem that seemed intractable at first. Then the Aha! moment that made it trivial.

Inserting the right triangles in the semicircle as shown in Figure 1, we see the setup for the geometric mean, namely,

$$\frac{b}{s} = \frac{s}{a} \Rightarrow s^2 = ab$$

since all the right triangles are similar. And thus we immediately have the answer that the two areas are equal.

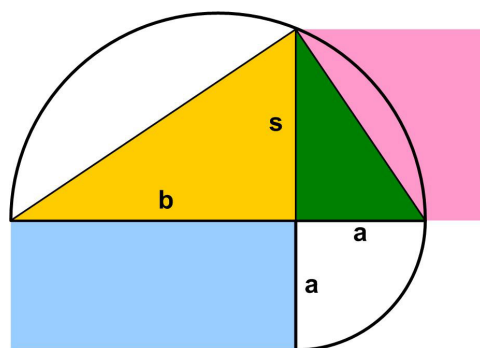


Figure 1

## Maths Masters Solution

**Answer:** The areas are equal.

**Solution:** There are three right-angled triangles hiding in the picture. They tell us that

$$a^2 + b^2 = c^2$$

$$b^2 + d^2 = e^2$$

$$c^2 + e^2 = (a + d)^2$$

Adding the three equations and cancelling, it follows that  $2b^2 = 2ad$ , and so the rectangle and square have the same area.

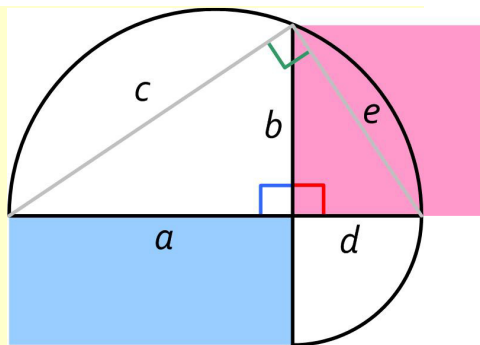


Figure 2

## References

- [1] Polster, Burkard and Marty Ross, “The Maths Masters' Summer Quiz, Problem Hard 3”, *The Age*, 9 December 2013 <https://www.qedcat.com/summerquizzes/2013%20QUIZ.pdf>

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