Alcuin's Corn Problem

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Alcuin of York (735-804) had a series of similar problems involving the distribution of corn among servants. Since the three propositions were the same format with only the numbers changing, I thought I would present them in a more concise form ([1] Props 32, 33, 34):

Proposition

A certain head of household had a number of servants, consisting of men, women, and children, among whom he wished to distribute quantities, modia, of corn. The men should receive three modia; the women, two; and the children, half a modium.

(a) If the head of household has 20 servants and wished to distribute 20 modia of corn among them, let him say, he who can, How many men, women and children must there have been.

(b) If the head of household has 30 servants and wished to distribute 30 modia of corn among them, let him say, he who can, How many men, women and children must there have been.

(c) If the head of household has 100 servants and wished to distribute 100 modia of corn among them, let him say, he who can, How many men, women and children must there have been.

I will give Alcuin's solutions first, followed by my more expansive solutions that rely on our familiar symbolic algebra that was not available in Alcuin's time.

Alcuin's Solutions

Alcuin's "solutions" are really just the answers, which I have summarized more succinctly following each of his "solutions". I will present a procedure for solving the problems below.

Solution (a)

Take one three times which makes three; that is, each man received this many modia. Likewise, take five twice, making 10; in this way, five women received 10 modia. Then, take two seven times, making 14; thus, 14 children received seven modia. Add one and five and 14, making 20; this is the number of servants. Then, add three and seven and 10, this being the number of modia. Thus there are 20 servants and 20 modia [of corn].

So Alcuin's answer for (a) is 1 man, 5 women, and 14 children.

Solution (b)

If you take thrice three, you get nine; if you take two five times, you get 10; and if you take half of 22, you get 11. Thus, three men received nine modia; five women received 10; and 22 children received 11 modia. Adding three and five and 22 makes 30 servants. Likewise, nine and 11 and 10 makes 30 modia. Hence there are 30 servants, and 30 modia [of corn].

So Alcuin's answer for (b) is 3 men, 5 women, and 22 children.

Solution (c)

11 times three makes 33, and twice 15 makes 30; that is, 11 men received 33 modia [of corn]. 15 women received 30 [modia], and 74 children received 37. Adding these together, that is, 11 and 15

and 74, makes 100, which is the number of servants. Likewise, adding 33 and 30 and 37 makes 100, which is the number of modia. Thus with these sums, you have 100 servants, and 100 modia [of corn].

And Alcuin's answer for (c) is 11 men, 15 women, and 74 children.

My Solutions

I make no apologies for using modern notation, since my object is not to subject anyone to duplicating ancient procedures, but rather to appreciate the power of symbolic algebra.

One of the questions left dangling from Alcuin's solution, other than how he got it, if not by trial and error, was whether there are any other solutions.

So we let

$$m = \text{number of men},$$

$$w = \text{number of women, and}$$

$$c = \text{number of children.}$$

Then for problem (a) we have

$$20 \text{ servants} = m + w + c$$

$$20 \text{ modia corn} = 3m + 2w + (1/2)c$$

Eliminating c, we have

$$20 = 3m + 2w + (20 - (m + w))/2$$

or

$$20 = 5m + 3w$$

or

$$w = 5\,(4 - m) / 3 \qquad (1)$$

Now, build a table of possible integral values for m, w, and c.

Men m	Women w	Children c
1	5	14

This is the only result possible, since in equation (1) we must avoid negative numbers and 4 – m has to be divisible by 3. Therefore, the answer to problem (a) is unique.

For the solution to problem (b), replace the number 20 by the number 30. This yields the equation

$$w = 5\,(6 - m) / 3 \qquad (2)$$

Now the table becomes

Men m	Women w	Children c
3	5	22

Again, this is the only solution to avoid negative numbers and have 6 – m be divisible by 3. (We are also assuming there are some each of men, women, and children.)

Finally, for the solution to problem (c), replace the number 30 by the number 100. This yields the equation

$$w = 5\,(20 - m) / 3 \qquad (3)$$

Now the table becomes

Men m	Women w	Children c
2	30	68
5	25	70
8	20	72
11	15	74
14	10	76
17	5	78

In this case we have multiple solutions to the problem. Alcuin only chose one of them, 11, 15, and 74. Alcuin did not explain why he chose this result and ignored the others.

References

[1] Burkholder, Peter J., *Alciun of York's "Propositiones ad Acuendos Juvenes"* (*"Propositions for Sharpening Youths"*), 1993. (https://www.math.muni.cz/-sisma/alcuin/anglicky1.pdf).