Stimulating Sequence

13 October 2022

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This is another stimulating little problem from the 2022 Math Calendar ([1]).

$$a_1 = 1,\ a_2 = 2,\ \ldots,\ a_{n+1} = a_n + 6a_{n-1}$$

$$x = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

Solve for x.

As before, recall that all the answers are integer days of the month.

Solution

Let $b_n = a_{n+1}/a_n$. Then

$$b_n = \frac{a_{n+1}}{a_n} = \frac{a_n + 6a_{n-1}}{a_n} = 1 + 6\frac{a_{n-1}}{a_n} = 1 + \frac{6}{b_{n-1}}$$

Assuming the limit exits, then $x = \lim_{n\to\infty} b_n$ implies

$$x = 1 + 6/x$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3.$$

References

[1] Rapoport, Rebecca and Dean Chung, *Mathematics 2022: Your Daily epsilon of Math*, Rock Point, Quarto Publishing Group, New York, 2022. December