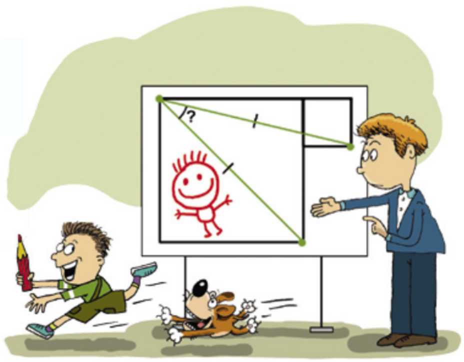
Two Squares Problem

10 August 2022

Jim Stevenson



Via Alex Bellos I found another Russian math magazine with fun problems. It is called *Kvantik* and Tanya Khovanova[1] has a description (2015):

Kvant[2] [*Quantum*] was a very popular science magazine in Soviet Russia. It was targeted to high-school children and I was a subscriber. Recently I discovered that a new magazine appeared in Russia. It is called *Kvantik*,[3] which means Little Kvant. It is a science magazine for middle-school children. The previous years' archives are available online in Russian. I looked at 2012, the first publication year, and loved it.

Unfortunately, the magazine is in Russian and the later issues are only partially given online. To get the full magazine you need to subscribe. I used Google Translate and the mathematical context to render the English. Here is an interesting geometric problem that I would have thought to be quite challenging for middle schoolers ([1]).

The vertices of the two squares are joined by two segments, as in the figure. It is given that these segments are equal. Find the angle between them.

Egor Bakaev

Solution

Assume the common length of the two lines is 1. Then the side of the large square is $1/\sqrt{2}$ (Figure 1). Let x represent the side of the smaller square and θ_1 and θ_2 the angles to the lines as shown. Then the unknowns angle is $\theta_2 - \theta_1$. Now

$$\tan(\theta_2 - \theta_1) = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1} = \frac{1 - \dfrac{x}{x + 1/\sqrt{2}}}{1 + \dfrac{x}{x + 1/\sqrt{2}}} = \frac{1}{2\sqrt{2}x + 1}$$

and by the Pythagorean Theorem

$$\left(x + 1/\sqrt{2}\right)^2 + x^2 = 1.$$

Therefore,

$$2x^2 + \sqrt{2}x - \tfrac{1}{2} = 0$$

which gives a positive solution of

$$x = \frac{-\sqrt{2} + \sqrt{6}}{4}.$$

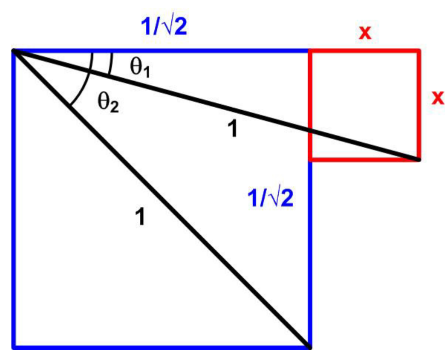


Figure 1

[1] https://blog.tanyakhovanova.com/2015/05/kvantiks-problems/
[2] https://en.wikipedia.org/wiki/Kvant_%28magazine%29
[3] https:/kvantik.com

And so $$\tan(\theta_2 - \theta_1) = \frac{1}{\sqrt{3}}$$

which means $$\theta_2 - \theta_1 = 30°$$

What a surprising, nice solution. That suggests there might be another, simpler way to get the answer, but I couldn't find one offhand.

References

[1] "Problem 49", *Kvantik*, No.6, June 2022 (https://kvantik.com/issue/pdf/2022-06_sample.pdf)