Mountain Climbing Puzzle

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This puzzle from the Scottish Mathematical Council (SMC) Middle Mathematics Challenge ([1]) has an interesting twist to it.

Two young mountaineers were descending a mountain quickly at 6 miles per hour. They had left the hostel late in the day, had climbed to the top of the mountain and were returning by the same route. One said to the other "It was three o'clock when we left the hostel. I am not sure if we will be back before nine o'clock." His companion replied "Our pace on the level was 4 miles per hour and we climbed at 3 miles per hour. We will just make it." What is the total distance they would cover from leaving the hostel to getting back there?

My Solution

When I first tried to solve the problem, I assumed the entire return trip was at 6 mph. But that led to an under-constrained problem with no specific solution. Then I realized the 6 mph was for the mountain section only and that the level section was still managed at 4 mph. This led to the space-time diagram in Figure 1.

In constructing the diagram I discovered that it did not matter how long the level stretch L was, since the total distance traveled D to the top of the mountain was the same. That meant the entire 6 hour trip could be done at the 4 mph rate, which would mean the total distance was 4 x 6 = 24 miles.

But I needed to prove this mathematically. So from Figure 1 we get the following equations:

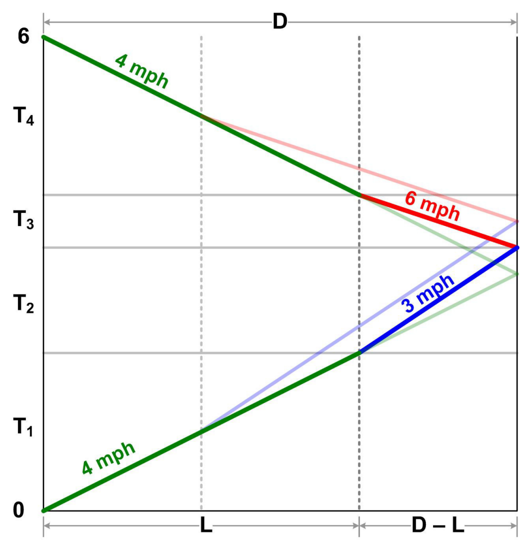

Figure 1

$$L = 4T_1$$

$$D - L = 3T_2$$

$$D - L = 6T_3$$

$$L = 4T_4$$

Multiplying the first equation by 3, the second by 4, the third by 2, the fourth by 3, and adding the four equations yields

$$(4 + 2)D + (3 + 3)L - (4 + 2)L = 12(T_1 + T_2 + T_3 + T_4)$$

or

$$6\,D = 12 \cdot 6$$

Therefore, D = 12 and the round trip is $2D = 24$ miles.

Notice that the level distance L disappeared from the computations so that it did not matter what it was. Still, I wanted a clearer explanation of what was happening on the mountain section. Viewing the situation from the point of view of the times, we had

$$T_2 + T_3 = (D - L)/3 + (D - L)/6 = (D - L)/2 = 2(D - L)/4$$

which implied

$$4(T_2 + T_3) = 2(D - L).$$

Therefore,

$$4(T_2 + T_3) = 3T_2 + 6T_3,$$

which is what I noticed from the diagram, that is, the mountain section could be eliminated and the whole trip taken at 4 mph and the distance would remain the same.

SMS Solution

The SMS solution ([2]) directly solves the problem from the point of view of the times, which is where I ended up in my analysis.

Let x be the number of miles covered ascending and y the number of miles on the level on the outward journey. So time taken to the summit is $x/3 + y/4$ hours. The time taken to get back is $x/6 + y/4$ hours. So the total time taken is the sum of these two which is $(x + y)/2 = 6$. So $x + y = 12$ and the total distance was 24 miles.

References

[1] "Middle Division: Problems 1 M3" *Mathematical Challenge 2006−2007*, The Scottish Mathematical Council (http://www.wpr3.co.uk/MC-archive/M/M-0607-Q1.pdf)

[2] "Middle Division: Problems 1 Solutions M3" *Mathematical Challenge 2006−2007*, The Scottish Mathematical Council (http://www.wpr3.co.uk/MC-archive/M/M-0607-S1.pdf)