Shared Spaces Puzzle

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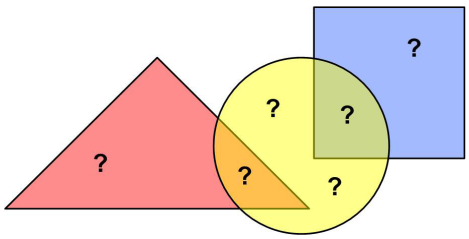


This is a nice puzzle from the Scottish Mathematical Council (SMC) Senior Mathematical Challenge of 2008 ([1]). It is more a logic puzzle than a geometric one.

In the diagram, each question mark represents one of six consecutive whole numbers. The sum of the numbers in the triangle is 39, the sum of those in the square is 46 and the sum of those in the circle is 85. What are the six numbers?

My Solution

The first step was to assign letters to the unknown numbers (Figure 1). Then the statement of the problem says

$$A + B = 39 \tag{1}$$

$$E + F = 46 \tag{2}$$

$$B + C + D + E = 85 \tag{3}$$

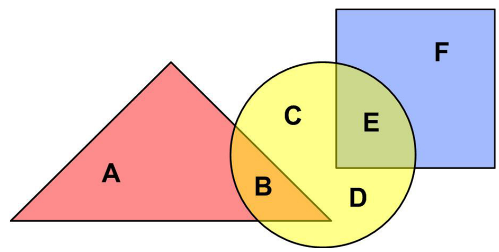


Figure 1

Adding equations (1) and (2), we get

$$A + B + E + F = 85$$

which, when subtracted from equation (3), gives the relations

$$A + F = C + D \tag{4}$$

We haven't used the fact that the numbers are consecutive whole numbers. So we proceed by trial and error, using the facts in equations (1) and (2).

18	19	20	21	22	23	✗	We don't have numbers big enough to add up to 46. (Eq. (2))
19	20	21	22	23	24	✓	OK
20	21	22	23	24	25	✗	We don't have numbers small enough to add up to 39. (Eq. (1))

Therefore we have the following possibilities:

A	B	C	E	D	F
B	A		F		E
19	**20**	**21**	**22**	**23**	**24**

By elimination, C and D must be 21 and 23, and the order does not matter. So C + D = 44. From equation (4) this means A + F = 44, which can only happen if A = 20 and F = 24. Therefore B = 19 and E = 22 (Figure 2).

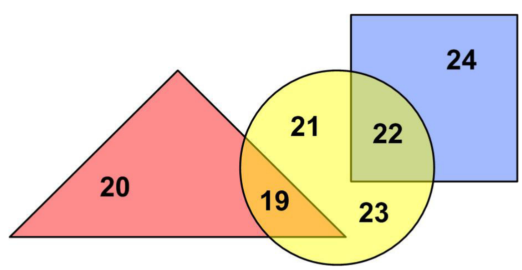


Figure 2

SMC Solution

The SMC solutions ([2]) avoid a trial and error guess at the consecutive numbers. But the reasoning is more convoluted.

Let the consecutive numbers be a, $a + 1$, $a + 2$, $a + 3$, $a + 4$, $a + 5$.

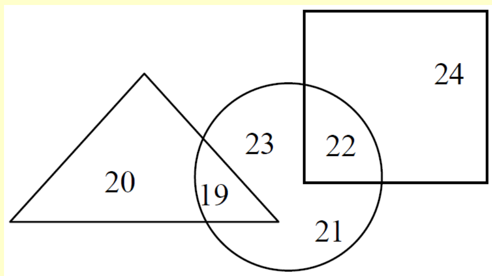
Method 1.

The sum of the two numbers in the square less the sum of the two numbers in the triangle is 7. There are two ways this could arise. Either $a + 4$ and $a + 5$ are in the square and a and $a + 2$ are in the triangle or $a + 3$ and $a + 5$ are in the square and a and $a + 1$ are in the triangle.

But the sum of the two numbers in the square is even so they must be $a + 3$ and $a + 5$. So $2a + 8 = 46$, giving $a = 19$. So the four numbers in the circle are $a + 2$, $a + 4$ and one of $a + 3$, $a + 5$ and one of a, $a + 1$ i.e. they are 21, 23 and one of 22, 24 and one of 19, 20. Since their sum is 85 they must be 21, 23, 22, 19. Thus the numbers are distributed as shown below.

Method 2.

Let the number in the intersection of the circle and triangle be $a + t$ and the number in the intersection of the circle and the square be $a + s$. So the total of the six numbers is $6a + 15$ and that is equal to $39 + 46 + 85 – (a + t) – (a + s)$. So $6a + 15 = 170 – (2a + s + t)$. So $8a = 155 – (s + t)$. Now $(s + t)$ is between 1 and 9 and so $8a$ is between 154 and 146. Since a is a whole number this means that $a = 19$. Furthermore with $a = 19$ we get $s + t = 3$. Now the sum of the two smallest numbers is 39 so they must be in the triangle. The sum of the numbers in the square is 46 so they must be 22 and 24. So we must have $s = 3$ and $t = 0$. This gives which numbers lie in the intersections and hence all numbers as shown below.



References

[1] "Senior Division: Problems 2" *Mathematical Challenge 2007−2008*, The Scottish Mathematical Council (http://www.wpr3.co.uk/MC-archive/S/S-2007-08-Q2.pdf)

[2] "Senior Division: Problems 2 Solutions" *Mathematical Challenge 2007−2008*, The Scottish Mathematical Council (http://www.wpr3.co.uk/MC-archive/S/S-2007-08-S2.pdf)