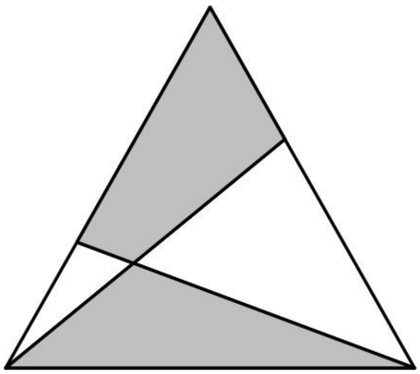
Sizing Up

25 July 2020

Jim Stevenson



This is another fairly simple puzzle from *Futility Closet* ([1]) from a while ago (2014).

Two lines divide this equilateral triangle into four sections. The shaded sections have the same area. What is the measure of the obtuse angle between the lines?

My Solution

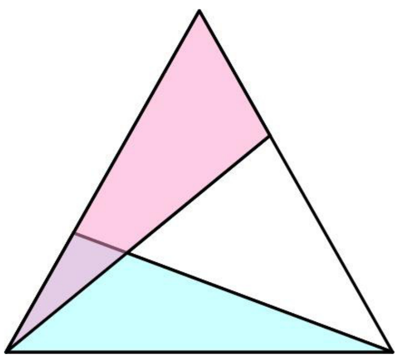


Figure 1 Equal Area Triangles

First notice that the colored triangles in Figure 1 have the same area, because the non-overlapped areas are the same from the problem statement and they share a common area in the overlap.

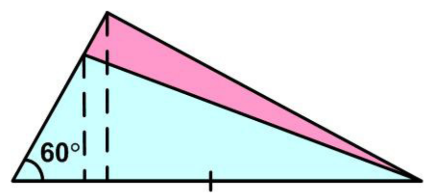


Figure 2 Congruent Triangles

This means the two triangles are congruent (Figure 2). This follows because they share a common angle (60°) and a common side or base (equilateral triangle). For their areas to be the same, they must have a common altitude as well. From that and trigonometry we can derive that the other two sides are equal and so the triangles are congruent.

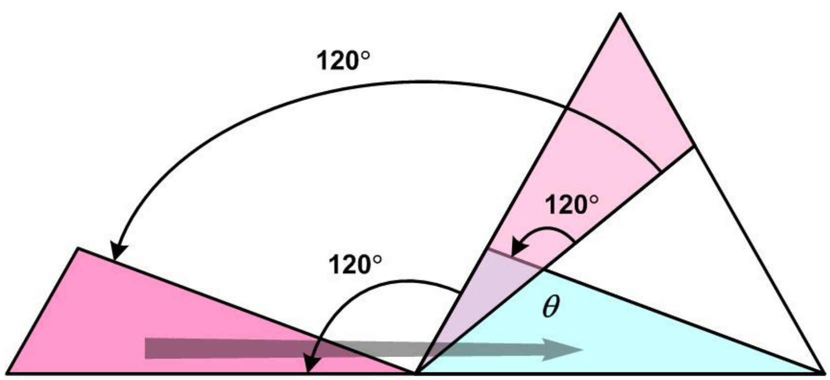


Figure 3 Rotated Triangles

This means we can rotate the pink triangle to lie along the base of the equilateral triangle and then slide it to the right over the blue congruent triangle (Figure 3). The rotation is 180° – 60° = 120°, which corresponds to the obtuse angle θ.

Futility Closet Solution

Triangles CAM and ABN have the same area. Rotating triangle ABC 120° takes CAM into ABM1. Since ABN and ABM1 have the same area and both N and M1 fall on BC, N = M1. Since rotating CM produces AN, the angle between them is 120°, or 180° – 120° = 60°, which is the same.

(Posed by V. Proizvolov in Math Horizons, Spring 1994.)

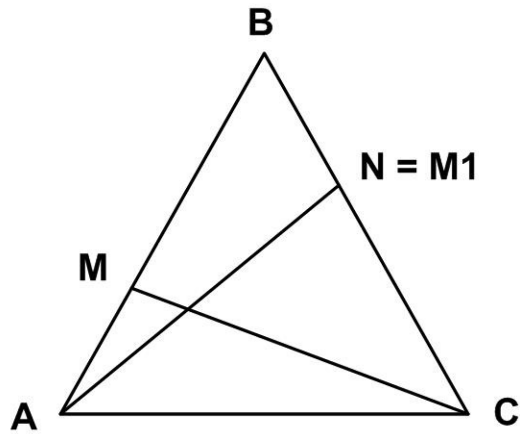
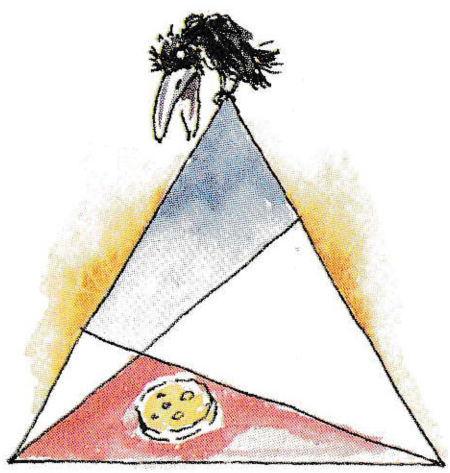


Figure 4 Futility Closet Solution

Comment.

Proizvolov's problem was also in *Quantum* ([2]) and the Futility Closet solution is the same as the one given in *Quantum*. As I did in my own argument (see Figure 2), I think there needs to be some explicit justification for claiming the two triangles with the same area are congruent.

Here is the figure from the *Quantum* statement of the problem:



Art by Pavel Chernusky

References

[1] "Sizing Up," *Futility Closet*, 25 November 2014 (http://www.futilitycloset.com/2014/11/25/sizing-up/, retrieved 6/19/2015)

[2] "Cutting an equilateral triangle", B105 "Brainteasers" *Quantum* Vol.4 No.3, Jan-Feb 1994. p.11