Turning Wheels Puzzle

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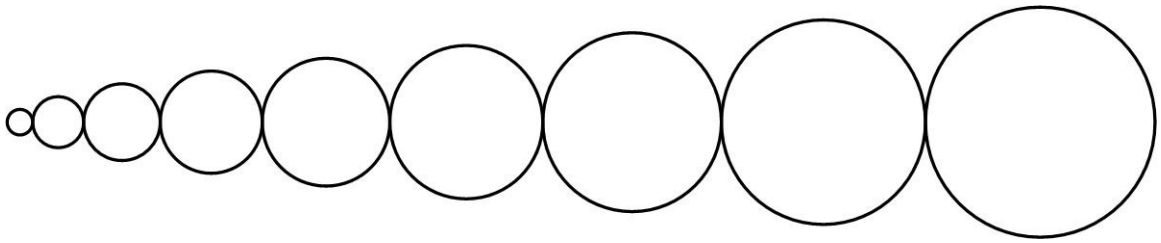

This is a thoughtful little problem from Posamentier's and Lehmann's *Mathematical Curiosities* ([1]).

We have nine wheels touching each other with diameters successively increasing by 1 cm. Beginning with 1 cm as the smallest circle, and 9 cm for the largest circle, how many degrees does the largest circle turn when the smallest circle turns by 90°?

My Solution

Let the radius of the i^{th} circle be r_i cm and the angle it turns θ_i radians. Then each circle must turn the same amount of its circumference as the initial circle, so

$$r_1\theta_1 = r_2\theta_2 = \ldots = r_9\theta_9$$

Therefore, $$2r_1\theta_1 = 2r_9\theta_9$$

and so $$1\cdot\pi/2 = 9\theta_9$$

or $$\theta_9 = \pi/18 = 180°/18 = 10°$$

Thus the 9^{th} circle turns 10°.

Posamentier's Solution

My solution is essentially the same as Posamentier's, only with more steps filled in.

A point on the circumference of a circle with diameter d that rotates α degrees will turn $\frac{\alpha}{360}\pi d$, where the circumference is πd. To determine the motion that we require here of α degrees is $\frac{90}{360}\pi\cdot 1 = \frac{\alpha}{360}\pi\cdot 9$, which gives us α = 10°.

References

[1] Posamentier, Alfred S. and Ingmar Lehmann, *Mathematical Curiosities: A Treasure Trove of Unexpected Entertainments*, Prometheus Books, 2014