Clock Connections Puzzle

31 January 2022

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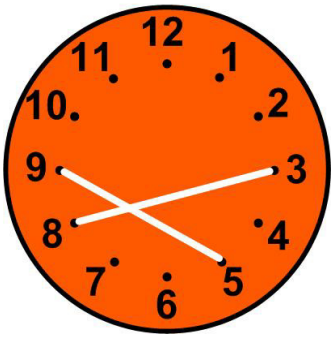

This is an imaginative puzzle from the Maths Masters team, Burkard Polster (aka Mathologer) and Marty Ross ([1]) as part of their "Summer Quizzes" offerings for 2012.

You draw a line connecting the 5 and 9 on a clock face, and another line connecting the 3 and 8. What is the angle between the two lines?

My Solution

The arrangement in the problem immediately reminded me of a generalization of the inscribed angle relation to the central angle, namely, where chords cross inside a circle rather than on its boundary, as shown in Figure 1.

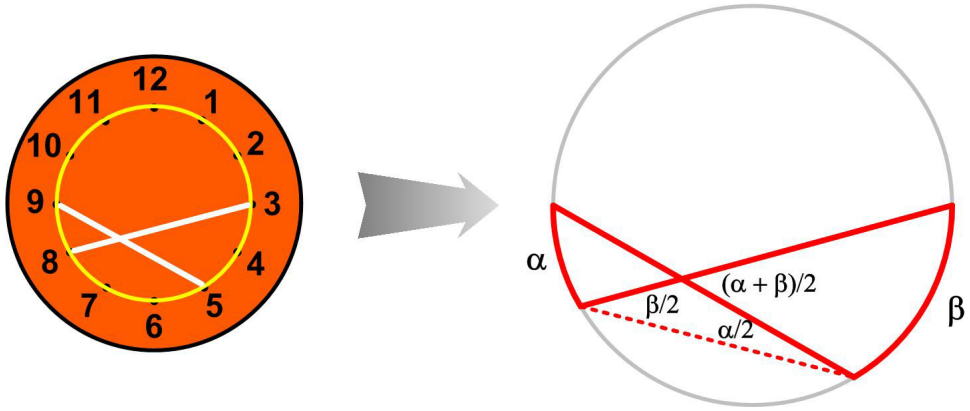

Figure 1

In our problem, the hour numbers on the clock are separated by 30°, so the corresponding central angles are $\alpha = 30°$ and $\beta = 60°$, so that the angle of interest is $(\alpha + \beta)/2 = 45°$.

Maths Masters Solution

The Maths Masters' solution is slick (Figure 2), though it depends on the particular choice of numbers connected, whereas our solution is general.

Answer: 45 degrees.

Solution. The two lines are parallel to the two ~~short~~ sides of the highlighted right-angled triangle. This triangle is clearly half of a square, and so the angle in question is 45 degrees.

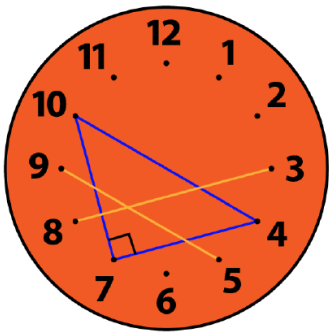

Figure 2

References

[1] Polster, Burkard and Marty Ross, "The Maths Masters' Summer Quiz, Problem Hard 2", *The Age*, 12 November 2012 (https://www.qedcat.com/summerquizzes/2012%20QUIZ.pdf)

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