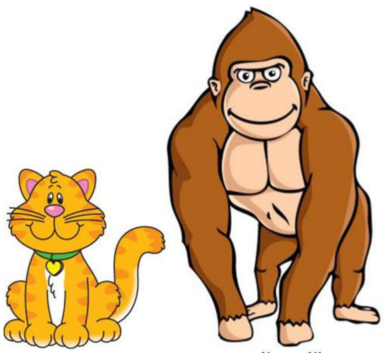
Heron Suit Problem

1 October 2020

Jim Stevenson



clipart-library.com

Here is another logic problem from Ian Stewart ([1]).

1. No cat that wears a heron suit is unsociable.
2. No cat without a tail will play with a gorilla.
3. Cats with whiskers always wear heron suits.
4. No sociable cat has blunt claws.
5. No cats have tails unless they have whiskers.

Therefore:

No cat with blunt claws will play with a gorilla.

Is the deduction logically correct?

I confess I don’t know what a heron suit is. Google showed various garments with herons imprinted on the cloth, so maybe that is what it is.

My Solution

We proceed by translating the sentences into symbolic logic notation. Assign the letters to statements as shown in the following:

H = A cat wears a heron suit.	G = A cat plays with a gorilla.
S = A cat is sociable.	W = A cat has whiskers.
T = A cat has a tail.	C = A cat has blunt claws.

Now we translate the given statements in the problem into those assigned above, remembering that the contrapositive of an implication $P \Rightarrow Q$ is $\sim Q \Rightarrow \sim P$. (Don’t confuse the contrapositive with the converse: $Q \Rightarrow P$.) To review symbolic logic constructs, see previous postings of logic puzzles.[1]

Statements	Implication	Contrapositive
1. If a cat wears a heron suit, then the cat is sociable.	$H \Rightarrow S$	$\sim S \Rightarrow \sim H$
2. If a cat has no tail, then the cat will not play with a gorilla.	$\sim T \Rightarrow \sim G$	$G \Rightarrow T$
3. If a cat has whiskers, then the cat wears a heron suit	$W \Rightarrow H$	$\sim H \Rightarrow \sim W$
4. If a cat is sociable, then the cat does not have blunt claws.	$S \Rightarrow \sim C$	$C \Rightarrow \sim S$
5. If a cat has a tail, then the cat has whiskers (A cat may not have a tail and either have whiskers or not have whiskers. But what cannot happen is a cat have a tail and not have whiskers. This is equivalent to $\sim(T \wedge \sim W)$, which is logically equivalent to $T \Rightarrow W$—they have the same truth table.)	$T \Rightarrow W$	$\sim W \Rightarrow \sim T$
∴ If a cat has blunt claws, then the cat will not play with a gorilla.	$C \Rightarrow \sim G$	$G \Rightarrow \sim C$

[1] http://josmfs.net/2020/09/19/pointing-fingers/, http://josmfs.net/2020/10/03/swallowing-elephants/, https://josmfs.net/2020/12/05/do-i-avoid-kangaroos/

Begin with C and try to form a chain of implications (⇒) ending in ~G.

$$C \Rightarrow \sim S \text{ (4 contra)} \Rightarrow \sim H \text{ (1 contra)} \Rightarrow \sim W \text{ (3 contra)} \Rightarrow \sim T \text{ (5 contra)} \Rightarrow \sim G \text{ (2)}$$

And so the deduction is logically correct. Statement 5 was the hardest to translate.

Stewart Solution

Now we have the surprising conclusion from Stewart:

The deduction is incorrect. Consider a cat with blunt claws that plays with a gorilla, does not wear a heron suit, has a tail, has no whiskers, and is unsociable. The first five statements are all true, but the sixth is not.

I have highlighted the phrase that violates my translation of statement 5. According to my interpretation, the highlighted statement is $T \wedge \sim W$, which is precisely what I said could *not* happen. That is, that statement must be false, or equivalently, $\sim(T \wedge \sim W)$ must be true, and $\sim(T \wedge \sim W)$ is logically equivalent to $T \Rightarrow W$. Therefore Stewart's counterexample is false and so his example does not invalidate the argument.

In fact, in surfing the web I found I was not alone in challenging Stewart's answer. In a thread at *The Museum of HP Calculators* Dave Britten also questioned Stewart's answer ([2]) with the same reasoning I employed. He was supported by EdS2, who cited some symbolic logic lecture notes by Venanzio Capretta ([3]) that confirmed Britten's and my proof—in a bit more labyrinthine form.

Comment

I can tell from the generally low visit numbers to posts involving logical problems such as this, that these problems are not particularly popular. In a way, I can sympathize. But ever since I tried solving them with symbolic logic, I found them somewhat interesting and challenging. Of course, the main issue is how to translate the "English" into symbolic logic phrases. This is the heart of the difficulty with all math word problems, however.

These "nonsense" logical puzzles probably elicit a greater feeling of real world irrelevance than most problems, but the sad irony for me is that there turned out to be a real-world application for the techniques. It is when we had to make out our wills with the "aid" of a lawyer. He phrased the set of conditions for various financial allotments based on a plethora of contingencies in such a way that I had to resort to symbolic logic to find out what was really happening. Hopefully my symbolic logic solution agreed with the lawyer's intention, but as we saw with some SCOTUS decisions, logic is not always high on their list.

References

[1] Stewart, Ian, "Heron Suit," *Professor Stewart's Cabinet of Mathematical Curiosities*, Basic Books, 2009. p.38

[2] "Heron Suit - Dave Britten - 05-30-2015 06:32 PM" at *The Museum of HP Calculators* (https://hpmuseum.org/forum/thread-4028.html)

[3] Capretta, Venanzio, *Mathematics for Computer Scientists,* Lecture notes for the module G51MCS, School of Computer Science, University of Nottingham (http://www.cs.nott.ac.uk/~vxc/g51mcs/ch01_logic.pdf)