# River Traffic Problem 

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Here is another elegant Quantum math magazine Brainteaser problem ([1]).

A raft and a motorboat set out downstream from a point A on the riverbank. At the same moment a second motorboat of the same type sets out from point B to meet them. When the first motorboat arrives at B, will the raft (floating with the current) be closer to point A or to the second motorboat? (G. Galperin)

## My Solution

Here is the pedestrian solution. In the space-time diagram (Figure 1) we label D the distance between A and $\mathrm{B}, \mathrm{D}_{1}$ the distance between A and the position of the raft, and $D_{2}$ the distance between the raft and the second motorboat. Let v be the speeds of both motorboats and $\mathrm{v}_{\mathrm{R}}$ the speed of the raft (or river). And let T be the time it takes the first motorboat to go from A to B.

Then we have the following equations


Figure 1 My Solution

$$
\begin{aligned}
\mathrm{D} & =\left(\mathrm{v}+\mathrm{v}_{\mathrm{R}}\right) \mathrm{T} \\
\mathrm{D}_{1} & =\mathrm{v}_{\mathrm{R}} \mathrm{~T} \\
\mathrm{D}-\left(\mathrm{D}_{1}+\mathrm{D}_{2}\right) & =\left(\mathrm{v}-\mathrm{v}_{\mathrm{R}}\right) \mathrm{T}
\end{aligned}
$$

The first two equations yield

$$
\mathrm{D}-\mathrm{D}_{1}=\mathrm{v} \mathrm{~T}
$$

which when combined with the third equation gives

$$
\begin{aligned}
\mathrm{D}-\left(\mathrm{D}_{1}+\mathrm{D}_{2}\right) & =\left(\mathrm{D}-\mathrm{D}_{1}\right)-\mathrm{D}_{1} \\
\mathrm{D}_{2} & =\mathrm{D}_{1}
\end{aligned}
$$

Or
So the raft will not be closer to either A or the second motorboat, but rather equidistant between them.

## Quantum Solution

The problem can be solved in the standard way by using equations of motion. Here's a more elegant solution.

Let C be the point upstream from A at the same distance from A as $\mathrm{B}(\mathrm{CA}=\mathrm{AB})$. Imagine a third motorboat that moves parallel with the first: it starts from C at the same speed, in the same direction, and at the same moment as the first motorboat. Since this third motorboat and the second motorboat move at the same speed relative to the raft (that is, to the water), both move toward the raft, and both are initially equidistant from the raft, they will always remain equidistant from the raft [Figure 2]. When the first motorboat arrives at B, the second [third?] covers the same distance and arrives at A. So at this moment the raft will be the same distance from the second motorboat as from point A (that is, the imaginary third motorboat). (V. Dubrovsky)


Figure 2 Quantum Solution

## References

[1] "River Traffic" B141 "Brainteasers" Quantum Vol. 5 No.5, National Science Teachers Assoc., Springer-Verlag, May-Jun 1995. p. 11
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