Tethered Triangle Puzzle

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This is another problem from MEI's MathsMonday.[1]

> Two equilateral triangles share a common vertex. Show that the lengths marked *a* and *b* are equal for any such arrangement.

This seems quite amazing at first. One can picture the small triangle swinging back and forth with red bungee chords tethering its bottom vertices to the bottom vertices of the large triangle. It would seem remarkable that the lengths of the chords would remain equal to each other throughout.
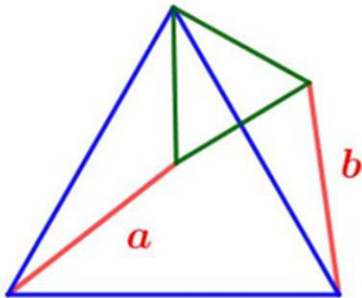
Solution

To find a solution we start considering the constraints of the problem: why two equilateral triangles? Well, they are similar and the small green triangle can be swung back so that its sides lie along the sides of the large blue triangle. Then we realize that as the small triangle swings to the right, its edges make the same angle with the sides of the large blue triangle (Figure 1). And so suddenly we have two congruent (yellow) triangles (SAS), which means the third sides *a* and *b* must be equal.
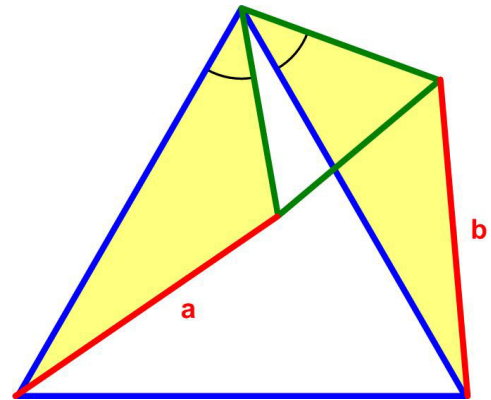
Figure 1

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[1] 6 July 2020 https://twitter.com/MEIMaths/status/1280111156171018249