Hard Time Conundrum

29 September 2021

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This problem comes from the "Problems Drive" section of the *Eureka* magazine published in 1955 by the Archimedeans at Cambridge University, England ([1]). ("The problems drive is a competition conducted annually by the Archimedeans. Competitors work in pairs and are allowed five minutes per question ….")

There are ten times as many seconds remaining in the hour as there are minutes remaining in the day. There are half as many minutes remaining in the day as there will be hours remaining in the week at the end of the day. What time is it on what day?

One of the hardest parts of the problem is just being able to translate the statements into mathematical terms. Solvable in 5 minutes?!!!

Solution

Let 0 be 12 midnight on Saturday night. Let d be the day where $d = 0$ is Sunday, $d = 1$ is Monday, …., $d = 6$ is Saturday. Let h be the hour in the day ($0 \le h < 24$), m the minute in the hour ($0 \le m < 60$), and s the second in the minute ($0 \le s < 60$) for the desired time. Then we translate the problem statements as follows:

Statement 1: $$60\cdot 60 - (60m + s) = 10(24\cdot 60 - (60h + m)) \quad (1)$$

Statement 2: $$24\cdot 60 - (60h + m) = \tfrac{1}{2}\,(7 - (d + 1))24 \quad (2)$$

The immediate difficulty is that we have four unknowns, d, h, m, s and only two equations. But we do have some constraints on the values that will provide some inequalities.

We will consider equation (2) first, since it involves the day d with only 7 choices. First, express the minutes m in terms of the hours h and days d.

$$m = 60(24 - h) - 12(6 - d) = 12(114 - 5h + d)$$

Then the constraint on the minutes $0 \le m < 60$ means we have the inequality

$$h > 21.8 + d/5$$

We present the choices in Table 1.

Table 1

d	$21.8 + d/5$	$< h < 24$	$m = 12(114 - 5h + d)$
0	21.8	22, 23	48, –12 X
1	22.0	23	0
2	22.2	23	12
3	22.4	23	24
4	22.6	23	36
5	22.8	23	48
6	23.0	X	X

Now we consider equation (1) in which we solve for seconds s in terms of minutes m and days d via equation (2).

$$s = 60(60 - m) - 10 \cdot 12(6 - d) = 60(48 - m + 2d)$$

With the constraint $0 \le s < 60$ we have the inequality

$$m > 47 + 2d$$

From the values in Table 1 the only viable possibility is $d = 0$ and $m = 48$. And so $s = 0$.

Thus, the answer for the time is $d = 0$, $h = 22$, $m = 48$, $s = 0$ or Sunday, 10:48:00 pm. (*Eureka* did not provide a method of solution—only the answer.)

References

[1] "Problems Drive", *Eureka*, The Journal of the Archimedeans, The Cambridge University Mathematical Society: Junior Branch of the Mathematical Association, No. 18, November 1955.