Equitable Slice Problem

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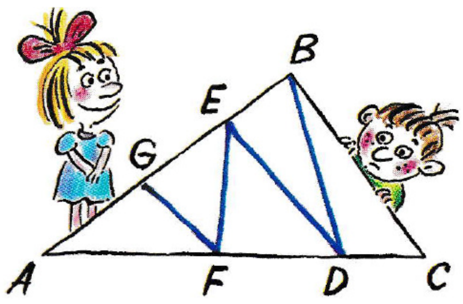
Jim Stevenson



Art by Edward Nazarov

This is another Brainteaser from the *Quantum* math magazine ([1]).

How can a polygonal line BDEFG be drawn in a triangle ABC so that the five triangles obtained have the same area?

I found this problem rather challenging, especially when I first tried to solve it analytically (using hyperbolas). Eventually I arrived at a procedure that would accomplish the result.

My Solution

My solution depends heavily on the area-preserving properties of reflections and the "shear" action on triangles. Figure 1 through Figure 4 show the steps to solving the problem. The first triangle uses the altitude of the big triangle and base 1/5 of AC, thus giving it an area of 1/5 of the large triangle (Figure 1). Subsequent equal triangles are constructed via reflections and shearing, thus preserving the 1/5 area of the larger triangle.

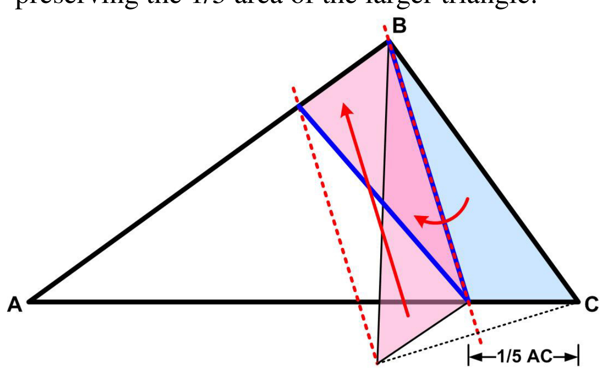


Figure 1 Lay out first triangle with base = 1/5 AC. Flip it around its side and then shear to the side of the large triangle.

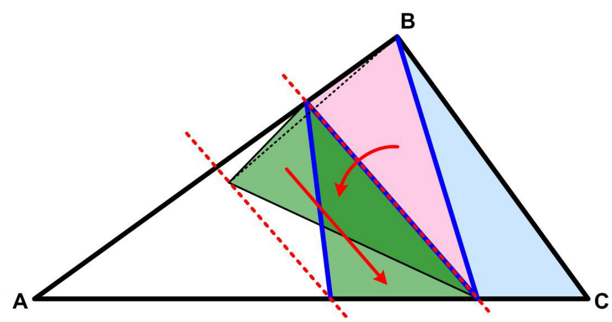


Figure 2 Flip second triangle around its side and shear result down to the base of the large triangle.

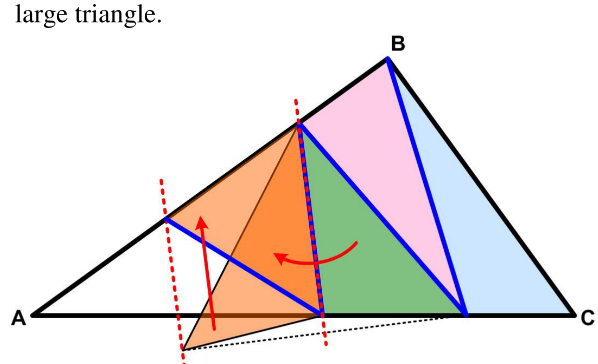


Figure 3 Flip third triangle around its side and shear result up to the side of the large triangle.

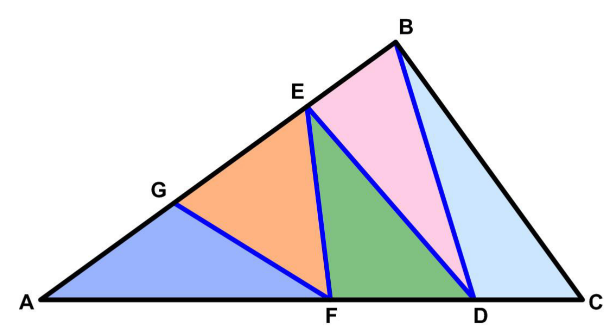


Figure 4 Remaining space (1 – 4/5 = 1/5) constitutes the fifth triangle of equal area.

Quantum Solution

Point D should be positioned so that segment CD is equal to l/5 of segment AC (Figure 5); then the area of triangle DBC will be 1/5 that of ABC. Similarly, point E is positioned so that BE = AB/4, point F so that FD = AD/3, and point G so that EG = AE/2.

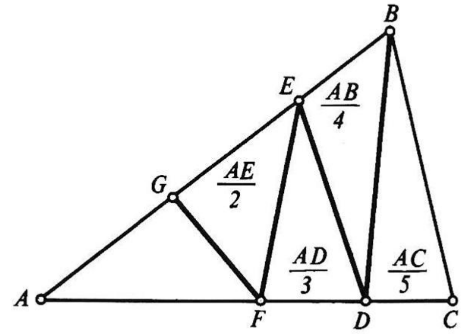


Figure 5 ***Quantum*** **Solution**

I originally misunderstood their approach and thought they had not solved the problem. It took me a little thinking to see what they were doing. The following figures show the steps behind the *Quantum* solution.

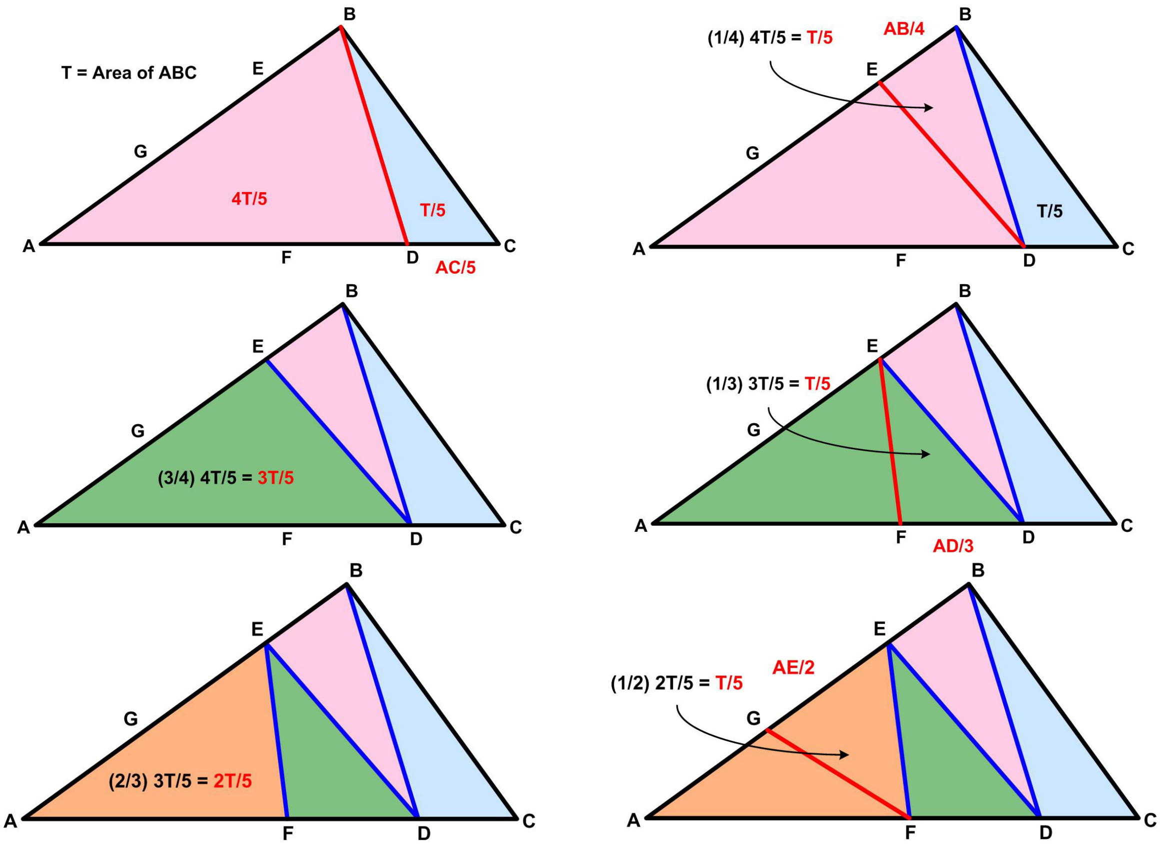


Figure 6

So finally (Figure 7) we arrive at the *Quantum* solution shown in Figure 5. It turns out to be a clever solution and provides a simple method to construct the answer.

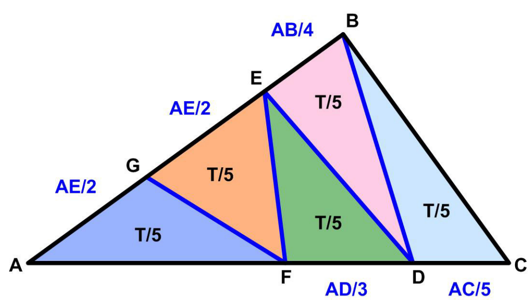


Figure 7

References

[1] "Equitable Slice", B11 "Brainteasers" *Quantum* Vol.1, No.1, National Science Teachers Assoc., Springer-Verlag, Sep-Oct 1990. p.19