Walking Banker Problem

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Jim Stevenson

Art by Pavel Chernusky

Here is another Brainteaser from the *Quantum* magazine ([1]).

Mr. R. A. Scall, president of the Pyramid Bank, lives in a suburb rather far from his office. Every weekday a car from the bank comes to his house, always at the same time, so that he arrives at work precisely when the bank opens. One morning his driver called very early to tell him he would probably be late because of mechanical problems. So Mr. Scall left home one hour early and started walking to his office. The driver managed to fix the car quickly, however, and left the garage on time. He met the banker on the road and brought him to the bank. They arrived 20 minutes earlier than usual. How much time did Mr. Scall walk? (The car's speed is constant, and the time needed to turn around is zero.) (I. Sharygin)

I struggled with some of the ambiguities in the problem and made my own assumptions. But it turned out there was a reason they were ambiguous.

My Solution

Figure 1 shows the setup where v_C is the speed of the car and v_B is the speed of the walking banker. T_0 represents the time it normally takes for the car to drive the banker a distance D_0 from his home to the bank. T is the time the banker walks until he is met by the car, and D is the distance he walked.

The ambiguities I encountered in the problem were when did the driver leave the garage to pick up the banker and where was the garage? So I assumed the garage was at the bank and I interpreted the statement that he left "on time" to mean he left at his usual time so that he could get to the house at his usual time. So that would be T_0 hours before picking up the banker.

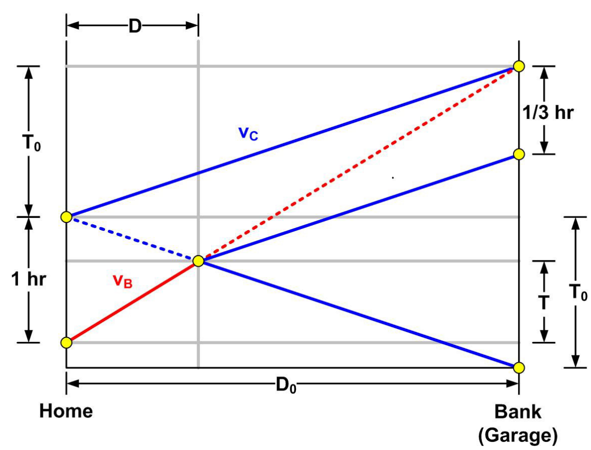

Figure 1 Spacetime Diagram of the Problem

This setup leads to the following equations:

$$v_C\, T_0 = D_0 \tag{1}$$

$$v_B\, (T_0 + 1) = D_0 \tag{2}$$

$$v_B\, T = D \tag{3}$$

$$v_C\, (T_0 - T + 2/3) = D_0 - D \tag{4}$$

From my assumptions:

$$v_C\, (2T_0 - 1/3) = 2(D_0 - D) \tag{5}$$

From equation (5) and doubling equation (4) we get

$$v_C\,(2T_0 - 1/3) = 2\,v_C\,(T_0 - T + 2/3) \qquad (6)$$

or

$$2\,v_C\,T = 5/3\,v_C$$

or

$$T = 5/6 \text{ hours} = 50 \text{ minutes}$$

Hmmm. That's curious. I didn't need the other equations, and a number of terms canceled in equation (6). The *Quantum* solution explains why.

Quantum Solution

It follows from the statement of the problem that the car met the bank president when it was 20/2 = 10 minutes from the banker's house. So the total time Mr. Scall spent walking is 50 minutes (he stopped walking 10 minutes before the time the car usually arrived, and he started 1 hour before the car's usual arrival time).

Using my spacetime diagram may help make this clear (Figure 2). The only thing that matters is the 20 minute gap between when the car meets the banker and when it would have been at the same spot in a normal journey. When and where the driver started turns out to be irrelevant. Only that the car's speed is the same in all situations is what is important, in order to make parallel lines in the diagram. This 20 minute gap at the meeting point is really what was contained in equations (4) and (5). And so they were sufficient to solve the problem.

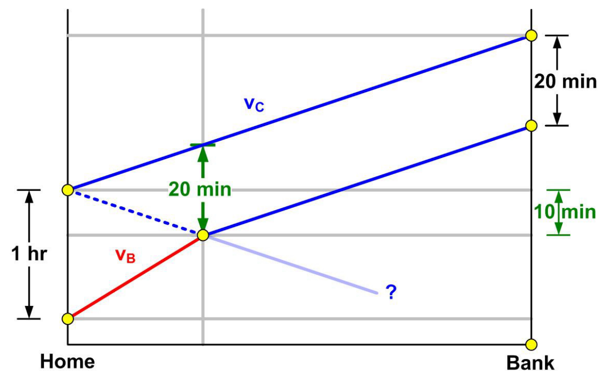

Figure 2 Quantum Solution

I of course did not realize this at first, and went through some hairy—and irrelevant—calculations involving equations (1) – (4), followed by using equation (5). I didn't realize at the time that it was only equations (4) and (5) that mattered. (And the fact that the T_0 terms canceled is an indication that the driver's start time and place did not matter, only that he got to the banker's house at the usual time.)

References

[1] "Pedestrian Banker" B208 "Brainteasers" *Quantum Magazine*, Vol.7 No.6, National Science Teachers Assoc., Springer-Verlag, Jul-Aug 1997 p.10