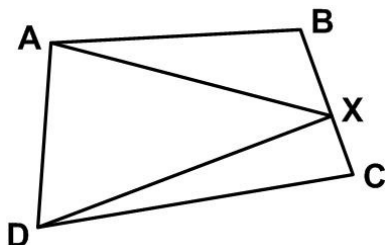


# Triangle Quadrangle Puzzle

22 February 2021

Jim Stevenson

This is another simple problem from *Five Hundred Mathematical Challenges* ([1]):



**Problem 57.** Let  $X$  be any point between  $B$  and  $C$  on the side  $BC$  of the convex quadrilateral  $ABCD$  (as in the Figure). A line is drawn through  $B$  parallel to  $AX$  and another line is drawn through  $C$  parallel to  $DX$ . These two lines intersect at  $P$ . Prove that the area of the triangle  $APD$  is equal to the area of the quadrilateral  $ABCD$ .

## Solution

Figure 1 shows the statement of the problem. We need to show the yellow triangle and blue quadrilateral have the same area. First notice both contain the (red) triangle  $AXD$  (Figure 2). So we only need to show the blue triangle  $ABX$  and yellow triangle  $APX$  have the same area. (The other pair of triangles,  $DXP$  and  $DCP$  are handled the same.) But since  $BP$  is parallel to  $AX$ , the blue and yellow triangles have the same altitudes (Figure 2). Since they also have the same bases, they have the same area. Done.

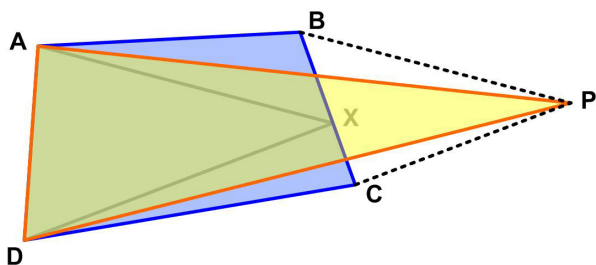


Figure 1

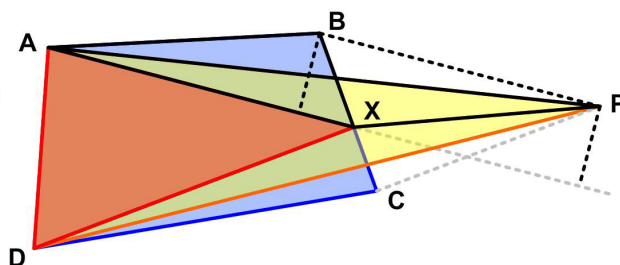


Figure 2

The 500 Math Challenges solution was the same.

## References

- [1] Barbeau, Edward J., Murray S. Klamkin, William O. J. Moser, *Five Hundred Mathematical Challenges*, Spectrum Series, Mathematical Association of America, Washington D.C, 1995

© 2021 James Stevenson