The Pearl Necklace Problem

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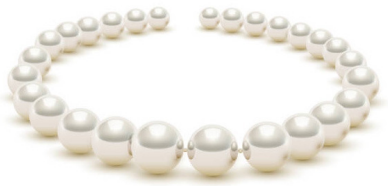

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This problem comes from the Scottish Mathematical Council (SMC) Senior Mathematical Challenge[1] of 2008 ([1]):

S2. In Tiffany's, a world famous jewellery store, there is a string necklace of 33 pearls. The middle one is the largest and most valuable. The pearls are arranged so that starting from one end, each pearl is worth \$100 more than the preceding one, up to [and including] the middle one; and starting from the other end, each pearl is worth \$150 more than the preceding one, up to [and including] the middle one. If the total value of the necklace is \$65,000 what is the value of the largest pearl?

I included the words in brackets to erase any ambiguity.

My Solution

Let V be the value of the most expensive, center pearl. Then the price of all the pearls in the necklace, working out on either side from the center one, is

\$65,000

$= (V - 16\cdot100) + (V - 15\cdot100) + \ldots + (V - 100) + \mathbf{V} + (V - 150) + \ldots + (V - 15\cdot150) + (V - 16\cdot150)$

$= 16V - (1 + 2 + \ldots + 15 + 16)100 + \mathbf{V} + 16V - (1 + 2 + \ldots + 15 + 16)150$

$= 33V - (16\cdot17/2)250$

$= 33V - 34{,}000$

Therefore $33V = \$99{,}000 \Rightarrow \mathbf{V} = \$3{,}000$

SMC Solution

Here is the SMC solution ([2]).

Let \$$m$ be the value of the first pearl at one end and \$$n$ the value of the pearl at the other end. So, counting from one end, the value of the middle pearl is $m + 16\times100$. Counting from the other end, the value of the middle pearl is $n + 16\times150$. Thus $m + 16\times100 = n + 16\times150$ so $m = n + 800$.

The total value of the necklace is the value of the middle pearl plus the other pearls whose value is

$m + (m + 100) + (m + 2\times100) + \ldots + (m + 15\times 100) + n + (n + 150) + (n + 2\times150) + \ldots + (n + 15\times150)$

$= 16\,(m + n) + (100 + 150)\,(1 + 2 + 3 + \ldots + 15)$

$= 16\,(m + n) + 30000.$

So the total value of the necklace is $16(m + n) + 30000 + m + 1600$. Eliminating m we get

$$16(2n + 800) + 30000 + n + 800 + 1600 = 65000.$$

[1] http://www.wpr3.co.uk/MC/ [JOS: the Scottish version of the UKMT Challenge]

Solving this equation for n we get $33n + 45200 = 65000$ which gives $n = 600$. Thus the value of the largest pearl is $600 + 16 \times 150 = 3000$.

References

[1] "Senior Division: Problems 1" *Mathematical Challenge 2007−2008*, The Scottish Mathematical Council (http://www.wpr3.co.uk/MC-archive/S/S-2007-08-Q1.pdf)

[2] "Senior Division: Problems 1 Solutions" *Mathematical Challenge 2007−2008*, The Scottish Mathematical Council (http://www.wpr3.co.uk/MC-archive/S/S-2007-08-S1.pdf)