Center of Square Problem

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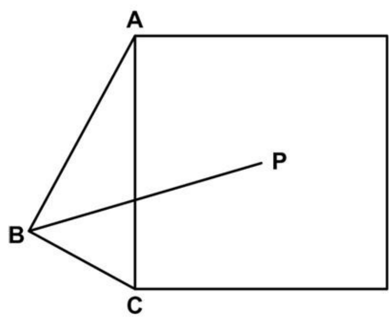


This is a simple problem from *Five Hundred Mathematical Challenges* ([1]):

Problem 24. Let P be the center of the square constructed on the hypotenuse AC of the right-angled triangle ABC. Prove that BP bisects $\angle ABC$.

My Solution

Since ABC is a right triangle, it can be inscribed in a circle of diameter AC or the hypotenuse of the triangle (Figure 1). The radius of this circle is half the length of the square, so the central point P lies on the circle at a right angle from A. This means the inscribed angle $\angle ABP$ must be half its corresponding central angle, or 45°. Since this is half the right angle at B, the line BP must bisect the angle $\angle ABC$.

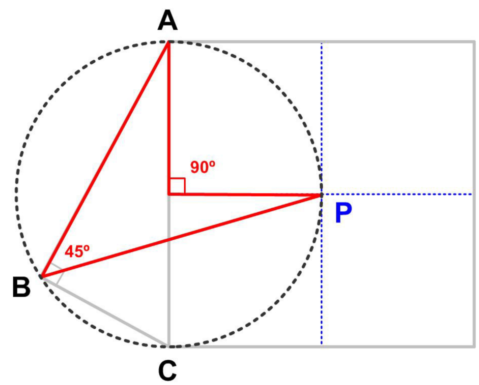
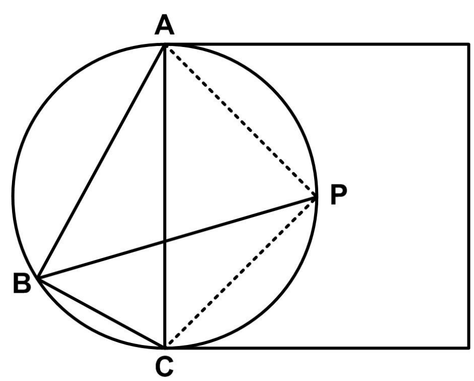


Figure 1

500 Math Challenges Solution.

Problem 24. Since $\angle APC$ and $\angle ABC$ are both right, the circle on diameter AC passes through B and P. Since AP and PC are equal chords of this circle they subtend equal angles at the circumference,[1] so $\angle ABP = \angle CBP$.



References

[1] Barbeau, Edward J., Murray S. Klamkin, William O. J. Moser, *Five Hundred Mathematical Challenges*, Spectrum Series, Mathematical Association of America, Washington D.C, 1995

[1] JOS: This is easy enough to show. Just make two congruent triangles from the central angles for these two arcs of the circle and their equal subtending chords. Then the central angles will be equal.